Massachusetts Institute of Technology
1.200J—Transportation Systems Analysis: Performance and Optimization
Fall 2015 — TA: Wichinpong "Park" Sinchaisri

Recitation 2
Unit 2 — Optimization Methodology

1 Intro to LP: Standard Form

Convert the following problems to the equivalent standard forms:

(a)

$$\begin{aligned} \underset{x_1,x_2}{\text{Maximize}} \quad & -2x_1 - 4x_2 \\ \text{subject to} \quad & x_1 + x_2 \geq 3, \\ & 3x_1 + 2x_2 \leq 14, \\ & x_1 \geq 0. \end{aligned}$$

(b)

$$\begin{aligned} \underset{x_1,x_2}{\text{Minimize}} \quad & 2x_1 + 3|x_2 - 10| \\ \text{subject to} \quad & |x_1 + 2| + |x_2| \leq 5. \end{aligned}$$

2 JetPurple's Marketing Plan

The world's newest airline, JetPurple, wants to focus its marketing to high-income women and men. To reach these groups, JetPurple launches an ambitious TV advertising campaign that will be aired on two types of programs: Kardashian-related reality shows and travel shows. Each reality commercial is seen by 7 million high-income women and 2 million high-income men, and costs \$50,000. Each travel commercial is seen by 2 million high-income women and 12 million high-income men, and costs \$100,000. JetPurple hopes to reach at least 28 million high-income women and 24 million high-income men.

(a) How can JetPurple meet its advertising requirements at minimum cost? Formulate this problem as an LP.

(b) Discuss the validity of the four LP modeling assumptions: (i) proportionality (ii) additivity (iii) divisibility, and (iv) certainty.

3 School District

Consider a school district with I neighborhoods, J schools, and G grades at each school. Each school j has a capacity of C_{jg} for grade g. In each neighborhood i, the student population of grade g is S_{ig}. Finally, the distance of school j from neighborhood i is d_{ij}.

Formulate an LP problem whose objective is to assign all students to schools, while minimizing the total distance traveled by all students.

4 Rocket Control

Consider a rocket that travels along a straight path. Let x_t, v_t, and a_t be the position, velocity, and acceleration, respectively, of the rocket at time t. By discretizing time and by taking the time increment to be unity, we obtain an approximate discrete-time model of the form.

$$x_{t+1} = x_t + v_t$$
$$v_{t+1} = v_t + a_t.$$

We assume that the acceleration a_t is under our control, as it is determined by the rocket thrust. In a rough model, the magnitude $|a_t|$ of the acceleration can be assumed to be proportional to the rate of fuel consumption at time t.

Suppose that the rocket is initially at rest at the origin. We wish the rocket to take off and land softly at unit distance from the origin after T time units. Furthermore, we wish to accomplice this in an economical fashion.

Formulate an LP problem to minimize the maximum thrust required.