1 Intro to LP: Standard Form

Convert the following problems to the equivalent standard forms:

(a) Maximize \( x_1 - 2x_1 - 4x_2 \)
subject to \( x_1 + x_2 \geq 3, \)
\( 3x_1 + 2x_2 \leq 14, \)
\( x_1 \geq 0. \)

(b) Minimize \( 2x_1 + 3|x_2 - 10| \)
subject to \( |x_1 + 2| + |x_2| \leq 5. \)

2 JetPurple’s Marketing Plan

The world’s newest airline, JetPurple, wants to focus its marketing to high-income women and men. To reach these groups, JetPurple launches an ambitious TV advertising campaign that will be aired on two types of programs: Kardashian-related reality shows and travel shows. Each reality commercial is seen by 7 million high-income women and 2 million high-income men, and costs $50,000. Each travel commercial is seen by 2 million high-income women and 12 million high-income men, and costs $100,000. JetPurple hopes to reach at least 28 million high-income women and 24 million high-income men.

(a) How can JetPurple meet its advertising requirements at minimum cost? Formulate this problem as an LP.

(b) Discuss the validity of the four LP modeling assumptions: (i) proportionality (ii) additivity (iii) divisibility, and (iv) certainty.
3 School District

Consider a school district with \( I \) neighborhoods, \( J \) schools, and \( G \) grades at each school. Each school \( j \) has a capacity of \( C_{jg} \) for grade \( g \). In each neighborhood \( i \), the student population of grade \( g \) is \( S_{ig} \). Finally, the distance of school \( j \) from neighborhood \( i \) is \( d_{ij} \).

Formulate an LP problem whose objective is to assign all students to schools, while minimizing the total distance traveled by all students.

4 Rocket Control

Consider a rocket that travels along a straight path. Let \( x_t, v_t, \) and \( a_t \) be the position, velocity, and acceleration, respectively, of the rocket at time \( t \). By discretizing time and by taking the time increment to be unity, we obtain an approximate discrete-time model of the form.

\[
\begin{align*}
    x_{t+1} &= x_t + v_t \\
    v_{t+1} &= v_t + a_t.
\end{align*}
\]

We assume that the acceleration \( a_t \) is under our control, as it is determined by the rocket thrust. In a rough model, the magnitude \( |a_t| \) of the acceleration can be assumed to be proportional to the rate of fuel consumption at time \( t \).

Suppose that the rocket is initially at rest at the origin. We wish the rocket to take off and land softly at unit distance from the origin after \( T \) time units. Furthermore, we wish to accomplish this in an economical fashion.

Formulate an LP problem to minimize the maximum thrust required.