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Markdown Pricing with Quality Perception and Consumer Optimism: From Experiment to Theory

Rim Hariss, Georgia Perakis, Wichinpong (Park) Sinchaisri, and Y. Karen Zheng

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Markdown Pricing with Quality Perception

Rim Hariss,[1] Georgia Perakis,[2] Wichinpong Park Sinchaisri,[3] Yanchong Zheng[2]
[1]Desautels Faculty of Management, McGill University, rim.hariss@mcgill.ca
[2]Sloan School of Management, Massachusetts Institute of Technology, georgiap@mit.edu, yanchong@mit.edu
[3]Operations, Information, and Decisions Department, The Wharton School, swich@wharton.upenn.edu

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Problem Definition: Consumers often perceive higher-priced products to have higher quality. Less is known on how quality perception is affected by price markdowns. In addition, it is an open question whether and how consumers' ex-ante expectation on a future markdown affects their quality perception as well as purchase decisions. We answer these questions in a markdown setting under a fixed inventory.

Academic/Practical Relevance: This paper adds to the growing literature that incorporates consumers' behavioral regularities in revenue management by studying the new dimension of quality perception and generates new insights absent in the current literature. Our results offer insights on how retailers should adapt their markdown strategy in the presence of price-based quality perception.

Methodology: We develop a consumer model that incorporates quality perception and emotional loss when the expected markdown is too optimistic as compared to the actual markdown. We embed this model into the retailer's markdown optimization and examine the impact of consumers' behavioral factors on the retailer's optimal strategy. Finally, we design and conduct a consumer study to calibrate our model and validate the functional relationships among key factors.

Results: Consumers' quality perception increases with the products full price while it decreases with the (expected) markdown. We show that the retailer's optimal markdown is nonmonotone in these quality perception parameters. The nonmonotonicity is driven by the nontrivial tradeoff of trying to maintain a higher perceived quality by the consumers while controlling potential loss emotion that could arise if consumers observe a smaller-than-expected markdown, particularly when total market demand is not very large. Furthermore, we find that it is beneficial for the retailer to pre-announce and commit to a markdown strategy to prevent a mismatch between consumers' expectation and the actual markdown. This approach benefits the retailer by eliminating the negative effect on sales of the consumers' loss emotion due to an optimistic expectation.

Managerial Implications: Ignoring these behavioral factors can substantially hurt the retailer's payoff. When inventory is tight, it is critical to correctly capture consumers quality perception (38% average loss in payoff if ignored). When instead inventory is sufficient, the retailer should be mindful of the potential emotional loss that its markdown could create among its consumers.

Keywords: Markdown pricing, quality perception, loss emotion, forward-looking consumers, behavioral operations

1. Introduction

Price and quality are two important dimensions that consumers consider when making a purchase decision. Interestingly, it has long been shown that the selling price of a product is a critical signal of product quality perceived by consumers (e.g., Monroe 1973, Rao and Monroe 1988, Gneezy et al. 2014). A higher price often induces higher quality perception (e.g., Rao and Monroe 1989, Rao and Bergen 1992, Bagwell and Riordan 1991, Lichtenstein et al. 1991, Plassmann et al. 2008).Capturing the price–(perceived) quality relationship is particularly relevant for fashion retailers who frequently practice price markdowns. Under a markdown strategy, the retailer sells a product at a high initial price and marks the price down as the product approaches the end of its selling season. Recent statistics show that sales under price markdowns have contributed to more than 30% of total revenues in department and specialty stores in the United States, up from less than 10% in the 1970s (Fisher and Raman 2010). Frequent practice of price markdowns has gradually changed consumers' purchase patterns, and such changes have caught the attention of both practitioners and researchers. We have seen a surge of studies in revenue management that investigate forward-looking consumer behavior (where consumers postpone their purchase to take advantage of markdowns) and the impact of such behavior on the retailer's optimal strategy (e.g., Besanko and Winston 1990, Aviv and Pazgal 2008, Gallego et al. 2008, Levin et al. 2009). However, this body of research has not examined the impact of markdowns on consumers' quality perception of the product and the associated purchase behavior.

Another behavioral impact of the frequent practice of markdowns is to induce consumers to form a certain expectation about the potential markdown that would be applied in the future. For example, if consumers frequently observe a 50% markdown in past seasons, then they may naturally expect the same markdown being applied in the current season. Researchers have shown that historical prices and discounts substantially influence consumers' reference prices for future transactions (e.g., Kalwani et al. 1990, Kalwani and Yim 1992, Briesch et al. 1997). The impact of consumers' expected markdown on their perceived quality of the product is yet to be known. In addition, consumers' expected markdown does not necessarily match the actual markdown applied. If the actual markdown is smaller than consumers' expectation, then consumers may experience an emotional loss for paying a higher-than-expected price. This emotional loss affects consumers' tendency to purchase the product at markdown. Therefore, it is important for the retailer to account for this emotional effectwhen devising its markdown strategy.

In this paper, we examine how consumers' price-based quality perception and loss emotion due to an optimistic markdown expectation impact their purchase decisions, and as a result, affect the retailer's optimal markdown strategy. To do so, we develop a markdown optimization model that incorporates both consumers quality perception and their potential loss emotion. We characterize the retailers optimal markdown strategy and analyze the operational as well as financial impacts of these behavioral factors. Furthermore, we design and conduct an online consumer study to validate the key behavioral factors modeled. Our results offer valuable insights on how a retailers optimal markdown strategy should be adapted in the presence of these behavioral factors.

Related Literature and Contributions: Our work is closely related to the recent stream of revenue management (RM) literature that focuses on developing richer and more realistic consumer models. A large group of researchers examine consumers' forward-looking behavior in various RM contexts, such as dynamic pricing (e.g., Elmaghraby et al. 2008, Li et al. 2014, Besbes and Lobel 2015, Harsha et al. 2016), promotions (e.g., Su 2010, Cohen et al. 2017), and capacity management (e.g., Liu and van Ryzin 2008, Cachon and Swinney 2009, Osadchiy and Vulcano 2010). See Shen and Su (2007) and Aviv and Vulcano (2012) for comprehensive reviews. More recently, researchers have begun to capture consumers' behavioral regularities beyond the full-rationality regime (see Ovchinnikov (2018) for a recent review). Some of the behavioral factors that have been studied include reference dependence (e.g., Popescu and Wu 2007, Heidhues and Kőszegi 2008, Nasiry and Popescu 2011, Baron et al. 2015, Tereyağoğlu et al. 2017), anticipated regret (e.g., Nasiry and Popescu 2012, Özer and Zheng 2016), availability misperception (Özer and Zheng 2016), time inconsistency (Su 2009, Baucells et al. 2016), and social comparison (Zhou et al. 2016). Recent experimental studies have investigated similar settings. For example, Osadchiy and Bendoly (2015) study whether consumers are forward-looking and find that consumers' decisions to wait for future discounts heavily depend on their perceived risk of future availability.

We contribute to this literature in two ways. First, to the best of our knowledge, we are the first in the RM field to *incorporate price-based quality perception into the optimization of markdown pricing strategies.* While a large body of studies exist in marketing that examine how price-based quality perception affects purchase intention (e.g., Rao and Monroe 1989, Raghubir and Corfman 1999, Suk et al. 2012, Gneezy et al. 2014), none of them formally analyze how pricing models should be adapted in light of such perception. We bridge this gap. In addition, following the well-established literature on reference pricing (e.g., Briesch et al. 1997, Popescu and Wu 2007), we also capture consumers' potential loss emotion due to a discrepancy between their expected markdown and the actual markdown being applied. Allowing consumers to have incorrect expectation of future markdown differs from the common assumption of rational expectations in previous RM models and captures more realistic scenarios. Our approach also follows calls for advancing consumer research by studying the joint impacts of multiple salient behavioral factors (Narasimhan et al. 2005, Ho et al. 2006).

Second, we *supplement the analytical modeling with an online consumer study that examines and validates the role of consumers' quality perception in affecting their purchase behaviors.* Within the behavioral operations management field, research that combines modeling with human-subject experiments and utilizes the experimental data to validate/calibrate operations models is only recently gaining attention (e.g., Becker-Peth et al. 2013, Scheele et al. 2018). The only papers related to RM that we are aware of employing such an approach are Ovchinnikov (2011) and Baucells et al. (2016). Closest to ours, Baucells et al. (2016) focus on consumers' risk and time preferences. We instead study a completely different dimension – consumers' price-based quality perception.

Our analysis yields a number of new insights. We consider a model for consumers' price-based quality perception where the perceived quality increases with the products full price while it decreases with the (expected) markdown. We show that the retailer's optimal markdown is nonmonotone in these quality perception parameters. The nonmonotonicity is driven by the nontrivial tradeoff of trying to maintain a higher perceived quality by the consumers while controlling potential loss emotion that could arise if consumers observe a smaller-than-expected markdown, particularly when total market demand is not very large. Furthermore, it is in the retailer's best interest to pre-announce and commit to a markdown strategy to prevent a mismatch between consumers' expectation about the markdown and the actual markdown applied. This approach benefits the retailer by eliminating the negative effect on sales of the consumers' loss emotion due to an optimistic expectation. Finally, we highlight that ignoring the behavioral factors captured in our model can substantially hurt the retailer's payoff. When inventory is tight, correctly capturing consumers' price-based quality perception has the most significant impact on the retailer's payoff (an average loss of 38% in payoff if ignored). When instead inventory is sufficient, the retailer should be particularly mindful of the potential emotional loss that its price markdown could create among its consumers.

2. Model Setup and Methodology

We consider a retailer who sells one product over two periods. The product is sold at its initial price in Period 1, and it may be marked down to a lower price in Period 2. A fraction of consumers arrive in Period 1. We call them "early consumers." The remaining fraction of consumers arrive in Period 2 and are called "late consumers." Early consumers observe the product sold at its initial price when they arrive in Period 1. They are aware that the retailer may apply a markdown to the product in Period 2; however, they do not know the level of markdown that will be applied. Early consumers choose among three options: (i) buying the product right away, (ii) waiting for the potential markdown and returning in Period 2, or (iii) leaving the market without buying. For those who wait and return in Period 2, they join the late consumers and observe the same product offered at either the initial price (if the retailer decides not to mark down) or at a discounted price (if the retailer decides to mark down). All consumers present in Period 2 either buy the product or leave the market without buying.

We model this setup as a bi-level optimization problem (Stackelberg game). The lower level models consumers' purchase decisions. Consumers need to decide whether and when to purchase the product to maximize their utilities. Both early and late consumers' utilities are affected by their quality perception of the product and the product's selling price. In addition, when early consumers arrive in Period 1, they form an expectation of the potential markdown in Period 2, and some of them may choose to wait for the markdown. The expected markdown creates a reference markdown price for these consumers. If the actual markdown is smaller than expected, then they may observe that the actual markdown is different from their expected markdown. When a discrepancy occurs, they experience emotional loss due to paying a higher-than-expected price. Such emotional loss eventually affects their purchase decision in Period 2. Our model of the consumers' quality perception and loss emotion is motivated by the literature and also validated based on data from an online consumer study that we design and conducted. In the upper level of the optimization problem, the retailer perfectly anticipates consumers' purchase decisions, and the retailer chooses the optimal markdown strategy based on this anticipation.

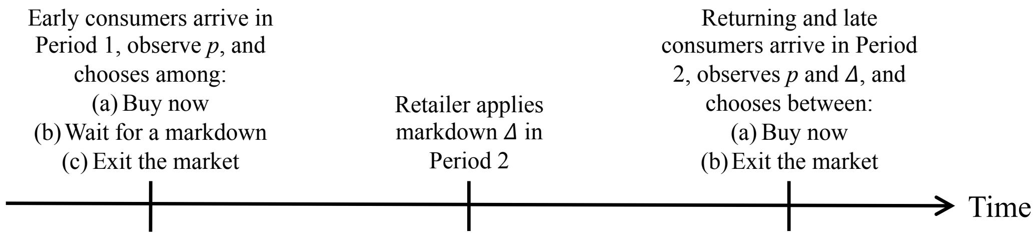
The remainder of the paper is structured as follows. In §3, we develop the bi-level optimization model and characterize consumers' optimal purchase decisions as well as the retailer's optimal markdown strategy. In §4, we examine and quantify the effects of ignoring the behavioral factors captured in our consumer model on the retailer's payoff. In §5, we discuss the design of an online consumer study to empirically validate and calibrate how consumers' price-based quality perception and their their potential loss emotion affect their purchase decisions.

3. The Model

We consider a retailer who sells a single product over two periods: Period 1 and Period 2. The retailer offers the product at its initial price p in Period 1, and it may apply a markdown $\Delta \in [0, p]$ in Period 2. A fraction $\gamma \in (0, 1)$ of consumers arrive in Period 1 (referred to as *early consumers*), and the remaining fraction $(1 - \gamma)$ of consumers arrive in Period 2 (referred to as *late consumers*). Our model of late consumers captures the presence of bargain hunters in the market, who only shop during markdown periods. We consider whether a consumer tends to be a bargain hunter to be mostly driven by their inherent shopping preferences, as opposed to specific operational factors. Prior studies (e.g., Cachon and Swinney 2009, Su 2009) have similarly assumed an exogenous fraction of late consumers in the market. The size of the market is normalized to 1. In Period 1, early consumers arrive to the market and observe the product sold at its initial price p. They know that the product may be marked down in Period 2, but they don't know the exact level of markdown that will be applied. Instead, they form some expectation of the potential markdown, denoted as $\tilde{\Delta}$. Early consumers choose among three options: (a) buy the product at price p, (b) wait for a markdown in Period 2, and (c) exit the market without buying the product. Those early consumers who choose to wait for a markdown and return to the market in Period 2 are hereafter called returning consumers. They join the late consumers and observe the product sold at price $p_2 = p - \Delta$. Both returning and late consumers choose between two options: (a) buy the product at price p_2, and (b) exit the market without buying the product. Each consumer's goal is to choose the purchase option that maximizes their utility. The retailer has a fixed inventory C of the product over both periods. If the total demand across both periods exceeds the inventory C, then the retailer incurs a cost s per unit of lost demand (underage cost). We take $s \geq p$ to capture potential intangible cost of missing demand beyond losing the revenue (e.g., customer dissatisfaction). The retailer's goal is to choose the optimal markdown Δ to maximize its total revenue in both periods minus the cost of underage. Figure 1 describes the timeline and decision sequence of our model.

Assumptions: We discuss here a few key assumptions in our model. First, the retailer focuses on optimizing its markdown decision while taking the initial price of the product as given. This assumption

Figure 1 Model Timeline



is motivated by the common practice in most fashion retailers that the decision on the initial price and the decision on markdown are made by different departments and managers, and that these decisions are rarely optimized jointly (e.g., Caro and Gallien 2012). Such separate optimization is also partly due to the large gap between the time when a retailer determines the initial price and the time when it makes markdown decisions. Second, we consider a fixed and exogenous inventory because fashion retailers typically need to make inventory decisions much earlier than the beginning of the selling season due to long production and transportation lead times. Thus, by the time the retailer needs to make markdown decisions, the inventory level is fixed and given. This assumption is widely adopted in the markdown pricing literature (e.g., Aviv and Pazgal 2008, Elmaghraby et al. 2008, Levin et al. 2009). Third, we model the retailer's inventory constraint following the approach of overbooking common in the revenue management literature (Talluri and van Ryzin 2004, Phillips 2005). This approach captures the first-order effect of limited inventory on the retailer's markdown decision without overcomplicating our model. Subtracting the underage cost from the retailer's revenue can be viewed as lagrangifying an explicit inventory constraint in the retailer's revenue maximization problem. Fourth, since there is no resolution of uncertainty between the two periods, analyzing the markdown decision at the start of Period 1 is equivalent to analyzing the markdown decision conditional on the remaining inventory at the start of Period 2. This is because the retailer can perfectly anticipate Period 1 demand by solving the consumer's utility maximization problem (the lower-level optimization problem).

In what follows, we first develop our consumer model, analyze early and late consumers' purchase decisions, and characterize the resulting market segmentation. Subsequently, we analyze the retailer's optimal markdown strategy, and determine how its optimal markdown decision as well as the resulting optimal payoff are affected by the consumers' behavioral factors.

3.1. The Consumer Model

We model consumers' utility as a combination of consumption utility and emotional utility with time discounting. Two key behavioral factors are captured: (i) consumers' quality perception of the product, and (ii) returning consumers' possible emotional loss due to the discrepancy between their expected markdown and the actual markdown applied. We model early and late consumers' quality perception q_1 and q_2 as follows:

$$q_1 = q_0 + ap - t\tilde{\Delta}, \tag{1}$$

$$q_2 = q_0 + a(p - \Delta). \tag{2}$$

The parameter q_0 reflects the underlying (objective) quality of the product determined by factors other than prices (e.g., the actual functionality or the brand). The parameter $a > 0$ captures the positive effect of the product's selling price on early and late consumers' quality perception, that a higher selling price is often associated with higher perceived quality (Rao and Monroe 1989, Rao and Bergen 1992, Bagwell and Riordan 1991, Lichtenstein et al. 1991, Plassmann et al. 2008). Prior studies also find that price discounts negatively affect consumers' quality perception (e.g., Raghubir and Corfman 1999). We extend this negative relationship to apply to early consumers' expected markdown (captured by the term $-t\tilde{\Delta}$ in Equation (1)). The rationale is the following. If a retailer has in the past frequently applied deep markdowns on its products, leading to a larger expected markdown by the consumers, then consumers likely perceive the retailer's products of lower quality, ceteris paribus. Finally, we assume that returning consumers would update their quality perception to follow Equation (2) after observing the actual markdown. In §5.2, we use data from a consumer study to validate our model of quality perception as discussed above.

Consumers' quality perception affects their purchase decisions through its impact on the consumption utility of buying the product. We capture this relation with a linear utility model commonly used in the literature (e.g., Choudhary et al. 2005, Chambers et al. 2006, Kalra and Li 2008). In particular, the consumption utilities of buying the product for early, returning, and late consumers can be respectively characterized as

$$U_c^e(\text{buy}) = \theta q_1 - p, \tag{3}$$

$$U_c^r(\Delta, \text{buy}) = U_c^l(\Delta, \text{buy}) = \theta q_2 - (p - \Delta), \tag{4}$$

with q_1 and q_2 defined in Equations (1) and (2). The superscripts e, r, and l indicate early, returning, and late consumers. The parameter θ captures consumers' heterogeneity in how much they value perceived quality against the selling price when making purchase decisions. A higher θ means the consumer values perceived quality more. The value θ is privately known to the consumer, and the retailer only knows that θ is uniformly distributed between 0 and $\theta_{\max}$ within the population of consumers (see Liu and van Ryzin 2008, 2011 for similar assumptions). We focus on the more interesting case that the consumer with the highest valuation earns a positive consumption utility if they purchase the product.

There is a vast literature showing that consumers often form internal reference prices which affect their purchase decisions (see Mazumdar et al. 2005, Özer and Zheng 2012 for reviews). Historical prices and markdowns are shown to significantly impact consumers' reference prices (e.g., Kalwani and Yim 1992, Briesch et al. 1997). In our context, early consumers do not know the actual markdown to be applied in Period 2 but instead, have in mind an expected markdown in Period 1. This expectation creates a reference markdown price that they anticipate in Period 2. When they return in Period 2 and observe smaller markdown than expected, they may experience a feeling of loss due to the higher-than-expected price. We model this emotional effect following the well-known phenomenon of loss aversion (e.g., Kahneman and Tversky 1979, Tversky and Kahneman 1992)[1].

We follow established models of reference-dependent preferences in the behavioral literature to capture consumers' potential loss emotion (e.g., Kőszegi and Rabin 2006, Popescu and Wu 2007, Ho et al. 2010, Nasiry and Popescu 2011). In particular, returning consumers' utility is the sum of a consumption utility and an emotional utility. The consumption utility, defined in Equation (4), captures the utility of consuming the product as in standard neoclassical economic theory. The emotional utility captures the loss emotion discussed above. As is common in the literature (e.g., Zeelenberg et al. 2000), we adopt a linear function for the emotional utility and model the strength of loss emotion as proportional to the gap between the actual and the expected markdowns. Thus, returning consumers' utility of buying the product in Period 2 is modeled as follows:

$$U^r(\Delta, \tilde{\Delta}, \text{buy}) = U_c^r(\Delta, \text{buy}) - \eta(\tilde{\Delta} - \Delta)^+, \tag{5}$$

where $U_c^r(\Delta, \text{buy})$ is defined in Equation (4) and $(x)^+ \equiv \max\{x, 0\}$. The parameter η captures the marginal value of the loss emotion relative to the consumption utility in returning consumers' final utility of buying the product. A higher value of η means that loss emotion has a stronger impact on the final utility to capture consumers' potential loss emotion.

Finally, when early consumers evaluate the option of waiting for a markdown, they discount their future utility by $\beta \in (0, 1]$. Such time discounting of utility can capture early consumers' patience in waiting for a markdown versus buying and owning the product immediately. A higher value means the consumer is more patient about waiting. Alternatively, time discounting can also be due to the loss in value of owning the product at a later time. This value loss is common for fashion products with a short season (e.g., Soysal and Krishnamurthi 2012). For example, swimming suits have a higher value at the beginning of summer than at the end. We do not distinguish between these two potential interpretations in our model, but simply refer to as early consumers' patience factor. In our consumer model, we treat the behavioral parameters q_0, a, t (in modeling quality perception), η (in modeling the marginal value of the loss emotion), $\tilde{\Delta}$ (the expected markdown), and β (the patience factor) as common knowledge and homogeneous across all consumers.

[1] The literature has shown that individuals are affected by both gain and loss emotions in decision making (Tversky and Kahneman 1992). In our setting, when returning consumers observe a larger markdown than expected, they may also experience a feeling of gain due to the lower-than-expected price. We verify that all of our results continue to hold if we also model this emotional gain in consumers' utility function. Furthermore, we confirm with the data from our consumer study that returning consumers do not exhibit gain emotion when observing a larger markdown than expected; however, they do exhibit loss emotion when observing a smaller markdown than expected (see §5.2.4).

3.2. Consumers' Purchase Decisions

Early consumers choose among three options: (a) buy the product in Period 1, (b) wait for a markdown in Period 2, and (c) exit the market without buying. If they choose to buy the product now, then they earn a utility of $U_c^e(\text{buy})$ defined in Equation (3). If they choose to wait, then they expect to obtain a utility of $U_c^w(\tilde{\Delta}, \text{buy}) = \theta q_1 - (p - \tilde{\Delta})$ when they buy the product in Period 2[2]. If they choose to exit, then they earn a utility of 0. When comparing between buying now and waiting, early consumers discount their expected utility of buying in Period 2 by β. Therefore, early consumers will choose to buy now if $U_c^e(\text{buy}) \geq \max\{\beta U_c^w(\tilde{\Delta}, \text{buy}), 0\}$; choose to wait if $\beta U_c^w(\tilde{\Delta}, \text{buy}) \geq \max\{U_c^e(\text{buy}), 0\}$; and choose to exit if $\max\{U_c^e(\text{buy}), \beta U_c^w(\tilde{\Delta}, \text{buy})\} < 0$. When early consumers who choose to wait return to the market in Period 2, they observe the actual markdown applied and choose between two options: buy the product or exit the market. If returning consumers choose to buy the product, then they earn a utility of $U^r(\Delta, \tilde{\Delta}, \text{buy})$ defined in Equation (5). Therefore, returning consumers will buy the product in Period 2 if $U^r(\Delta, \tilde{\Delta}, \text{buy}) \geq 0$. The following proposition characterizes early consumers' purchase decisions and the resulting segmentation of these consumers. All proofs are deferred to the appendix.

PROPOSITION 1 **(Segmentation of early consumers)**. *Define three thresholds $\underline{\theta} \leq \theta' \leq \bar{\theta}$ as*

$$\underline{\theta} \equiv \frac{p - \tilde{\Delta}}{q_1}, \quad \theta' \equiv \frac{p}{q_2} - \frac{\Delta - \eta(\tilde{\Delta} - \Delta)^+}{q_2}, \quad \bar{\theta} \equiv \frac{p}{q_1} + \frac{\tilde{\Delta}\beta}{q_1(1-\beta)}.$$

Let $\hat{\theta} \equiv \min\{\theta_{\max}, \bar{\theta}\}$, $\Delta_0 \equiv \frac{p + \eta\tilde{\Delta} - (q_0 + ap)\hat{\theta}}{1 + \eta - a\hat{\theta}}$, $\Delta_0^+ = \max\{0, \Delta_0\}$ *and* $\Delta_1 \equiv \max\{\frac{p + \eta\tilde{\Delta} - (q_0 + ap)\underline{\theta}}{1 + \eta - a\underline{\theta}}, \frac{p - (q_0 + ap)\underline{\theta}}{1 - a\underline{\theta}}\}$*. Then early consumers' purchase decisions are characterized as follows:*

(i) An early consumer buys the product at the initial price in Period 1 if their valuation of quality $\theta \in [\hat{\theta}, \theta_{\max}]$; waits for a markdown if $\theta \in [\underline{\theta}, \hat{\theta})$; and exits the market without buying if $\theta \in [0, \underline{\theta})$.

(ii) For returning consumers:

(a) If $\Delta \geq \Delta_1$, then all returning consumers buy the product at the markdown price in Period 2.

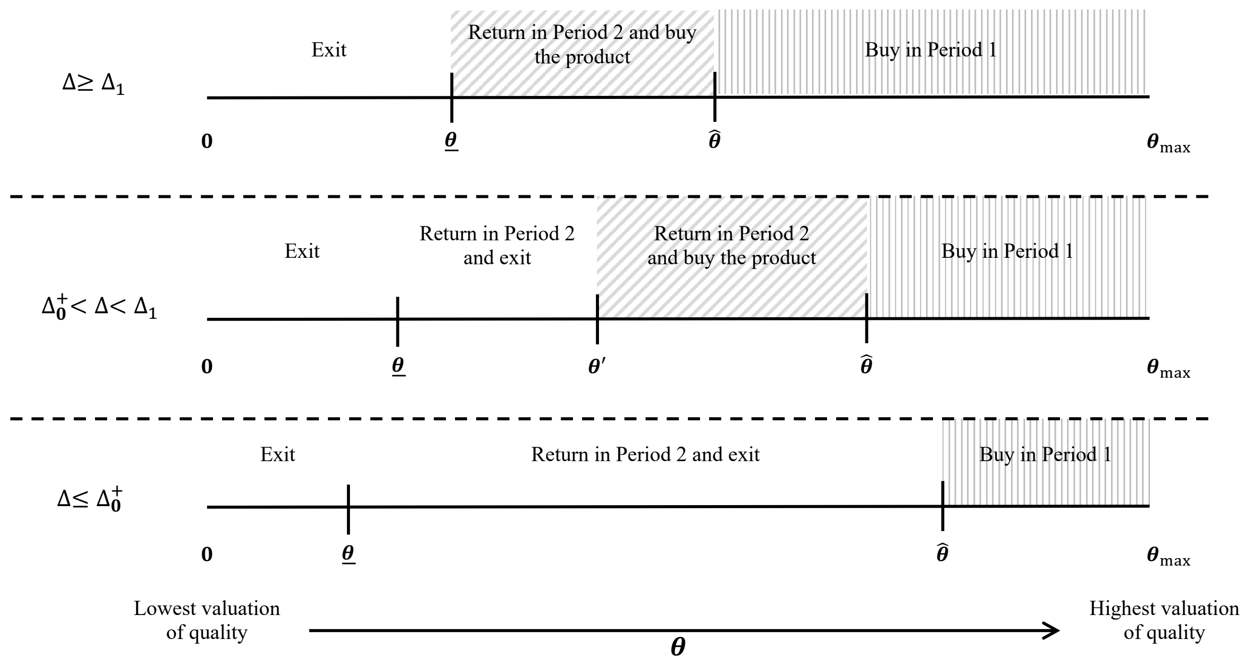
(b) If $\Delta \in (\Delta_0^+, \Delta_1)$, then a returning consumer buys the product at the markdown price if $\theta \in [\theta', \hat{\theta})$, and exits the market without buying if $\theta \in [\underline{\theta}, \theta')$.

(c) If $\Delta \leq \Delta_0^+$, then all returning consumers exit the market without buying the product.

Figure 2 illustrates the segmentation of early consumers given their valuations of quality, θ, and their expected markdown, $\tilde{\Delta}$, based on Proposition 1. Early consumers who highly value quality (with $\theta \in [\hat{\theta}, \theta_{\max}]$) buy the product at the initial price in Period 1 (the vertically shaded areas in Figure 2). Early consumers whose valuations are in the intermediate range ($\theta \in [\underline{\theta}, \hat{\theta})$) choose to wait for a markdown in Period 2. Finally, those early consumers who have a low valuation to begin with ($\theta \in [0, \underline{\theta})$) exit the market without buying.

[2] We make a simplifying assumption here that early consumers expect full availability of the product in Period 2 when evaluating the utility of waiting. This allows us to focus on examining quality perception without overcomplicating the analysis. To verify the robustness of our results, we numerically analyze an extension in which early consumers form rational expectations about product availability in Period 2. Our main insights remain unchanged in this extension. See Appendix B for more details.

Figure 2 Segmentation of Early Consumers (Proposition 1)



For early consumers who choose to wait for a markdown, whether they eventually purchase the product when they return in Period 2 depends on their expected markdown. If returning consumers had expected a smaller markdown than the actual one (i.e., $\Delta \geq \Delta_1$), then all of them would buy the product at the markdown price in Period 2. If they had expected a larger markdown than the actual one but the discrepancy is not too great (i.e., $\Delta \in (\Delta_0^+, \Delta_1)$), then only those with higher valuations ($\theta \in [\theta', \hat{\theta})$) would purchase the product at the markdown price, and the rest would exit the market without buying. In this case, the smaller markdown observed relative to the expected markdown results in a feeling of loss for returning consumers. Thus, the consumption utility from buying needs to be sufficiently high to compensate this loss emotion and motivate a returning consumer to buy the product. If returning consumers had expected a markdown that is much larger than the actual one (i.e., $\Delta \leq \Delta_0^+$), then none of them would buy the product in Period 2. This is because the loss emotion resulted from the overly optimistic expected markdown is so large that even the consumer with the highest valuation ($\theta = \hat{\theta}$) would not earn a positive utility from buying the product.

We now turn to analyzing late consumers' purchase decisions. Late consumers choose between two options: buy the product in Period 2 or exit without buying. Given late consumers' consumption utility $U_c^l(\Delta, \text{buy})$ defined in Equation (4), they will buy the product in Period 2 if $U_c^l(\Delta, \text{buy}) \geq 0$. The following proposition characterizes the resulting segmentation of late consumers.

Proposition 2 **(Segmentation of late consumers)**. *Define the threshold $\tilde{\theta}$ as $\tilde{\theta} \equiv \dfrac{p-\Delta}{q_2}$. A late consumer buys the product at the markdown price in Period 2 if their valuation of quality $\theta \in [\tilde{\theta}, \theta_{\max}]$, and exits without buying if $\theta \in [0, \tilde{\theta})$.*

Late consumers are segmented into two groups. Those with higher valuations ($\theta \in [\tilde{\theta}, \theta_{\max}]$) buy the product at the markdown price, and the rest exit without buying. Denote the fraction of early consumers who buy the product in Period 1 as D_1, the fraction of early consumers who buy the product in Period 2 as D_2, and the fraction of late consumers who buy the product in Period 2 as D_3. The following lemma discusses how these three demands change with the actual markdown Δ, the patience factor β, and the expected markdown $\tilde{\Delta}$.

LEMMA 1. *(i) D_1 is nonincreasing in β and $\tilde{\Delta}$.*

(ii) D_3 is increasing in Δ.

(iii) D_2 is nondecreasing in β and Δ. In addition, there exists $\tilde{\Delta}_1 \geq \Delta$ such that D_2 is increasing in $\tilde{\Delta}$ for $\tilde{\Delta} \in [0, \tilde{\Delta}_1]$ and decreasing in $\tilde{\Delta}$ for $\tilde{\Delta} \in (\tilde{\Delta}_1, p]$.

Lemma 1 parts (i) and (ii) are intuitive. The more patient early consumers are, a larger fraction of them would choose to wait for the markdown. Similarly, if early consumers expect a larger markdown, then they both perceive the quality of the product to be lower (see Equation (1)) and are more willing to wait for the markdown. Hence, D_1 decreases with β and $\tilde{\Delta}$. For late consumers, a larger actual markdown naturally increases demand. Thus, D_3 increases with Δ. The effects of β and Δ on the demand from returning consumers, D_2, follow similar reasoning as above. However, how the expected markdown $\tilde{\Delta}$ affects D_2 is less straightforward. Two counteracting forces coexist. On the one hand, a larger $\tilde{\Delta}$ motivates more early consumers to wait, hence increasing the potential demand in Period 2. On the other hand, a larger $\tilde{\Delta}$ implies lower perceived quality, thus leading to some early consumers exiting the market without buying. Furthermore, a larger $\tilde{\Delta}$ also means a higher chance for returning consumers to experience a loss when observing a smaller actual markdown, discouraging them from eventually buying. Lemma 1 part (iii) shows that the combined effects result in D_2 first increasing and then decreasing with respect to $\tilde{\Delta}$.

The retailer, being the first mover in the game, can fully anticipate consumers' purchase behavior described in Propositions 1 and 2. Therefore, the retailer takes into account consumers' purchase decisions when optimizing its markdown strategy. We next characterize the retailer's optimal markdown strategy and the conditions under which applying a markdown (versus not) is beneficial.

3.3. The Retailer's Optimal Markdown Strategy

The retailer chooses the level of markdown, Δ, to maximize its total revenue from both periods minus the potential cost of underage due to its inventory constraint. The retailer's objective is characterized as follows: $\Pi(\Delta) = \gamma p D_1 + \gamma(p-\Delta)D_2(\Delta) + (1-\gamma)(p-\Delta)D_3(\Delta) - s[\gamma(D_1 + D_2(\Delta)) + (1-\gamma)D_3(\Delta) - C]^+$. Here s is the underage cost per unit of lost demand, C is the total inventory of the retailer, and demands D_1, D_2, and D_3 are defined prior to Lemma 1. The following theorem characterizes the retailer's optimal markdown strategy.

Theorem 1 **(Retailer's optimal markdown strategy)**. *We can characterize valuation thresholds* $\theta_0 < \theta_1$ *and inventory threshold* $\underline{C}$ *such that the following results hold:*

(i) If $\theta_{\max} < \theta_0$*, then it is optimal for the retailer to always mark down in Period 2; i.e.,* $\Delta^* > 0$.

(ii) If $\theta_{\max} \in [\theta_0, \theta_1)$*, then it is optimal for the retailer to mark down in Period 2 (i.e.,* $\Delta^* > 0$*) if and only if* $C \geq \underline{C}$.

(iii) If $\theta_{\max} \geq \theta_1$*, then it is optimal for the retailer not to mark down in Period 2; i.e.,* $\Delta^* = 0$.

When the highest valuation among consumers, $\theta_{\max}$, is very low (i.e., $\theta_{\max} < \theta_0$ as in Theorem 1 part (i)), the retailer should always mark down in Period 2 to generate demand and increase revenue. When $\theta_{\max}$ is slightly higher (i.e., $\theta_{\max} \in [\theta_0, \theta_1)$ as in Theorem 1 part (ii)), it is optimal for the retailer to mark down as long as its inventory is not too low. With very low inventory, the retailer would rather have a small demand and minimize potential underage cost. Thus, not marking down is optimal. Finally, when $\theta_{\max}$ is very high (i.e., $\theta_{\max} \geq \theta_1$ as in Theorem 1 part (iii)), the potential market demand is so large that it is never optimal for the retailer to mark down in Period 2.

3.4. The Impact of Inventory

Proposition 3 describes how the retailer's optimal markdown, Δ^* and the resulting optimal payoff, Π^* are affected by the level of inventory, C, at the retailer.

Proposition 3 **(Impact of inventory on the retailer's optimal markdown and optimal payoff)**. *Given thresholds* θ_1 *and* $\underline{C}$ *defined in Theorem 1, the following results hold:*

(i) If $\theta_{\max} \geq \theta_1$*, then the optimal markdown* $\Delta^* = 0$ *and is independent of inventory* C.

(ii) If $\theta_{\max} < \theta_1$*, then* Δ^* *is nondecreasing in* C *when* $C \geq \underline{C}$.

(iii) Π^* *is nondecreasing in* C.

Proposition 3 part (i) follows directly from Theorem 1. In part (ii), when the consumers' highest valuation is not too high ($\theta_{\max} < \theta_1$) and the retailer's inventory is not too tight ($C \geq \underline{C}$), we observe the optimal markdown Δ^* weakly increase with respect to C. First, an increase in inventory motivates the retailer to apply a larger markdown in Period 2. This is because the retailer wants to (i) attract more demand to deplete the inventory, and (ii) mitigate the possible loss emotion of returning consumers when they compare the actual and their expected markdowns. In this range, the retailer makes its markdown decision to either exactly sell out all of its inventory (if the per-unit underage cost $s \geq p$). As the retailer's inventory level further increases, the goal of depleting all inventory is no longer financially desirable. Instead, the retailer focuses on maximizing sales revenue from both periods.

We next discuss the effect of the retailer's inventory level on its optimal payoff Π^*. When the retailer's inventory level is very tight, an increase in inventory improves its optimal payoff primarily due to a reduction in the underage cost incurred. As inventory increases, while the increased markdown reduces the retailers margin, the additional demand that it captures from both returning and late consumers more than compensates the reduced margin, thus resulting in a higher optimal payoff. Finally, when the inventory level is very large, the retailer has sufficient inventory to meet all the demand and thus applies a constant markdown that leads to the same optimal payoff irrespective of the inventory level.

3.5. The Impact of Quality Perception

We next analyze how consumers' quality perception impacts the retailer's optimal markdown Δ^* and payoff Π^*. Recall from Equations (1) and (2) that a captures the positive effect of the product's selling price on consumers' quality perception; t captures the negative effect of early consumers' expected markdown on their respective quality perception. The following result describes how the parameter a impacts the retailer's optimal markdown and optimal payoff.

PROPOSITION 4 **(Impact of a on the retailer's optimal markdown and optimal payoff).** *We can characterize four thresholds $a_1 \leq a_2 \leq a_3 \leq a_4$ such that the following results hold:*

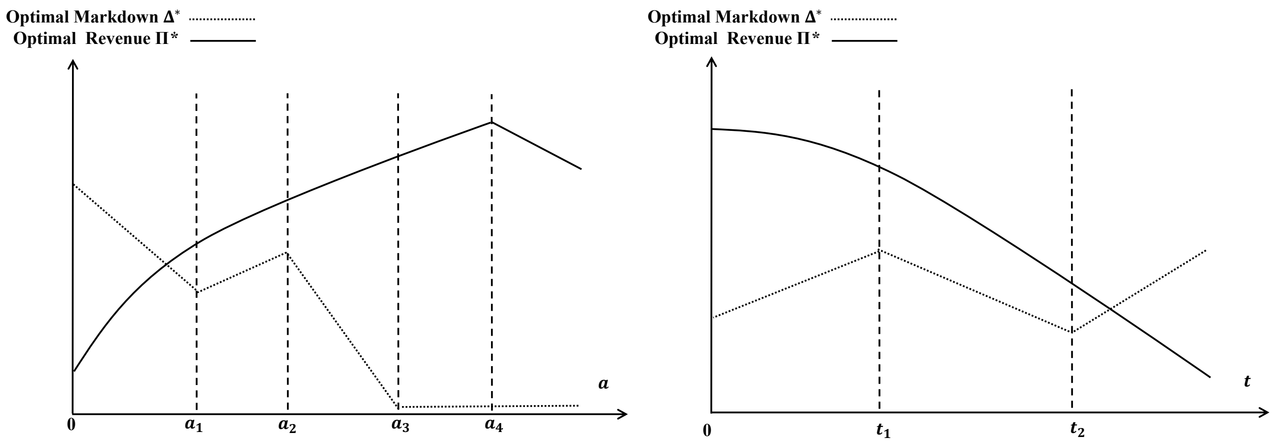
(i) If $a < a_1$, then Δ^* is nonincreasing in a. If $a \in [a_1, a_2)$, then Δ^* is nondecreasing in a. If $a \in [a_2, a_3)$, then Δ^* is nonincreasing in a. If $a \geq a_3$, then $\Delta^* = 0$ and is independent of a.

(ii) If $a \leq a_4$, then Π^* is nondecreasing in a. If $a > a_4$, then Π^* is nonincreasing in a.

Proposition 4 is illustrated in Figure 3a. We first focus on the impact of a on Δ^* (Proposition 4 part (i)). When a is very low ($a < a_1$), the retailer's optimal markdown decreases as a increases. With a low value of a, the total market demand is small, because consumers who overall have low quality perception. The retailer thus needs to apply a large markdown to attract consumers to buy. As a increases a little, the demand from early consumers (who either buy at the initial price in Period 1 or return and buy in Period 2) increases, and the retailer reduces the level of markdown to benefit from a larger margin. As a increases further but still not high ($a \in [a_1, a_2)$), the demand from returning consumers constitutes an important part of the retailers revenue. Thus, the retailer must ensure that it captures all the returning consumer demand while maintaining a good margin. Hence, it is optimal for the retailer to increase its markdown (as a increases) to counteract returning consumers loss emotion and motivate exactly all returning consumers to buy the product. When a is relatively high ($a \in [a_2, a_3)$), both returning and late consumers' consumption utilities are high, and they are willing to buy the product at a small markdown. Consequently, the retailer reduces optimally the level of markdown. Finally, when a is very high ($a > a_3$), the total market demand is so large that the retailer becomes more concerned about reducing the underage cost due to its inventory constraint. As a result, the retailer chooses not to mark down.

We next discuss the impact of a on the optimal payoff (Proposition 4 part (ii)). As a increases, consumers in general perceive higher quality for the product, implying a larger overall demand. The retailer thus earns a higher optimal payoff as long as total demand is less than the retailer's inventory. This pattern holds true until a becomes sufficiently high that the retailer no longer applies any markdown

Figure 3 **Impacts of Quality Perception Parameters (a and t on Optimal Markdown Δ^* and Payoff Π^* (Propositions 4 and 5)**



(a) Impact of a (b) Impact of t

Note. The dotted (solid) lines represent the retailer's optimal markdown (payoff).

in optimality. Thereafter, further increased values of a lead to the total demand exceeding the retailer's inventory. As a result, the retailer suffers from the underage cost, and its optimal payoff decreases.

The next result shows how the parameter t affects the retailer's optimal markdown and payoff.

PROPOSITION 5 **(Impact of t on the retailer's optimal markdown and optimal payoff)**. *We can characterize two thresholds $t_1 \leq t_2$ such that the following results hold:*

(i) If $t < t_1$, then Δ^* is nondecreasing in t. If $t \in [t_1, t_2)$, then Δ^* is nonincreasing in t. If $t \geq t_2$, then Δ^* is nondecreasing in t.

(ii) The optimal payoff Π^* is nonincreasing in t.

Figure 3b illustrates Proposition 5. When t is low ($t < t_1$), there is a significant portion of early customers that would wait for the markdown. The retailer initially applies a small markdown but as t increases, it has to apply a larger markdown to capture most of the demand from returning consumers. When t becomes higher ($t \in [t_1, t_2)$), the demand from returning consumers decreases but is still sizable. In this case, the retailer still tries to capture all of the returning demand but is able to reduce the level of markdown applied to generate a good size of demand from late consumers (recall that a markdown negatively affects late consumers quality perception). Finally, when t is very high ($t > t_2$), the overall demand from early consumers is very small. Therefore, the retailer needs to compensate the low sales from early consumers by increasing demand from late consumers. Thus, the retailer applies a larger markdown as t increases. Finally, the parameter t always negatively impacts the retailers optimal payoff. This result is primarily due to the negative effect t has on early consumers quality perception, and hence, their demand.

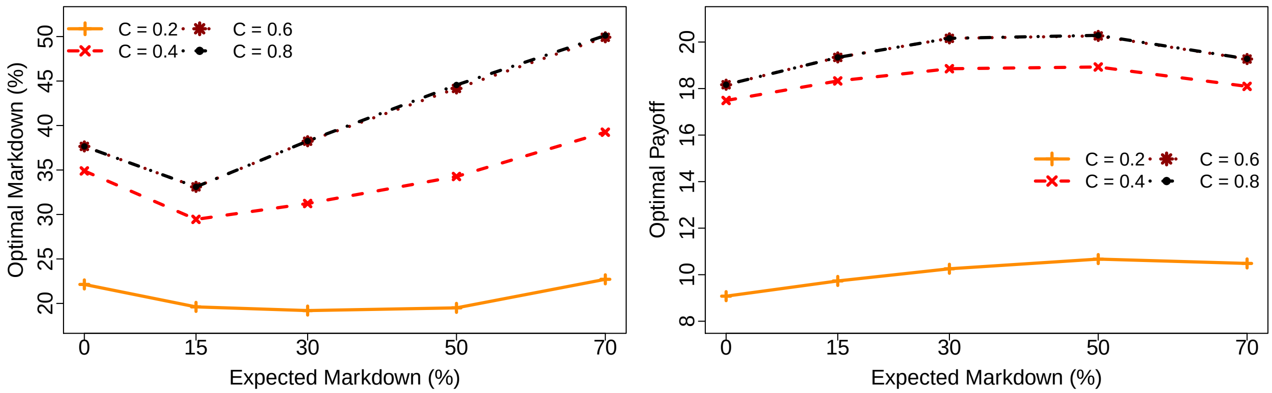
3.6. The Impact of Consumers' Expected Markdown

Here we examine how the retailer's optimal markdown and the resulting payoff are influenced by consumers' expectation about future markdown. We employ an extensive numerical analysis with the following set of parameter values: $\theta_{\max} = 1$, $p \in \{\$50, \$75, \$100\}$, $\gamma \in \{0.25, 0.5, 0.75\}$, $a \in \{0.15, 0.2, 0.225, 0.25, 0.275, 0.3, 0.35\}$, $t \in \{0.5a, 0.75a, 0.9a, a, 1.1a, 1.25a, 1.5a\}$, $q_0 \in \{0.5p, 0.75p, p, 1.25p, 1.5p\}$, $\eta \in \{0.25, 0.5, 0.75, 1\}$, $\tilde{\Delta}/p \in \{0, 0.15, 0.3, 0.5, 0.7\}$, $s \in \{p, 1.25p, 1.5p, 2p\}$, $C \in \{0.2, 0.4, 0.6, 0.8\}$, and $\beta \in \{0.25, 0.5, 0.75\}$. The values of a, t, and q_0 are chosen based on their estimated values from our consumer study (§5). In addition, the average expected percentage markdown reported by the participants in our consumer study is 30%. To perform our analysis, given a set of parameter values, we first randomly generate 1,000 consumers whose θ is uniformly distributed on $[0, \theta_{\max}]$ and compute the retailer's optimal markdown and payoff. We perform 100 simulations (with a different sample of 1,000 consumers in each simulation) for each parameter combination. This process results in approximately 280,00,000 numerical instances.

Figure 4a presents a representative example of how the retailer's optimal markdown is affected by the consumers' expected markdown, $\tilde{\Delta}$, for different inventory levels, C. We observe that the optimal markdown increases with $\tilde{\Delta}$ when $\tilde{\Delta}$ is not too small. This result is relatively straightforward. A larger $\tilde{\Delta}$ has two effects on the retailer's demand. First, early consumers' quality perception becomes lower (and hence, demand decreases). Second, early consumers are more likely to wait for a markdown, and further, there is a higher chance that they would experience a loss when observing the actual markdown in Period 2. Both of these effects motivate the retailer to apply a larger markdown to maintain revenue. However, when $\tilde{\Delta}$ is small (e.g., $\tilde{\Delta} \leq 15\%p$), we observe instead the optimal markdown being decreasing in $\tilde{\Delta}$. This contrasting pattern is due to the following dynamics. When early consumers almost do not expect a markdown to occur, they either purchase the product in Period 1 or exit the market. Hence, the markdown decision primarily affects demand from late consumers. As $\tilde{\Delta}$ increases, more and more early consumers choose to wait. As a result, the markdown decision affects revenue earned from both returning and late consumers. In this case, the retailer finds it beneficial to limit the extent of its markdown to maintain a relatively high margin per unit of demand satisfied in Period 2.

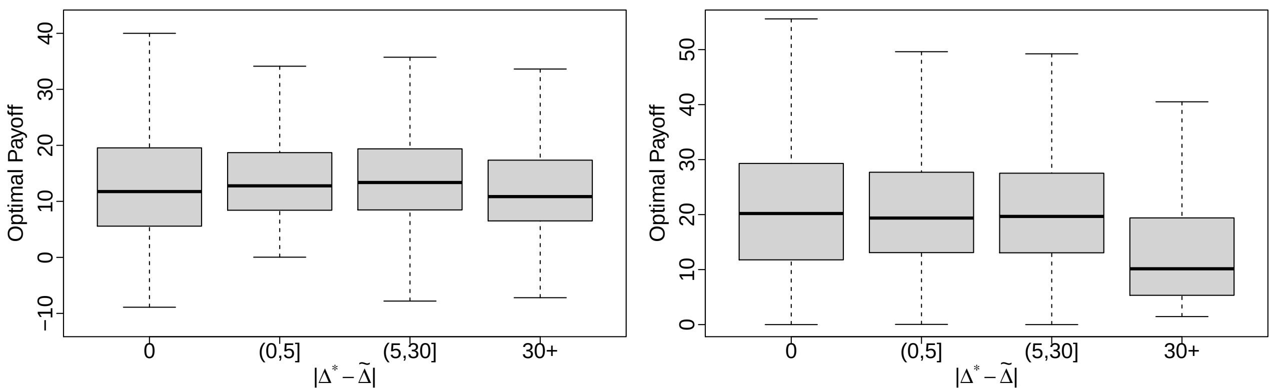
Figure 4b shows a representative example of how the retailer's optimal payoff changes with $\tilde{\Delta}$. We observe a general concave pattern; i.e., the retailer's optimal payoff first increases then decreases with $\tilde{\Delta}$. On the one hand, having a very small expected markdown leads to early consumers with lower valuations leaving the market instead of waiting for a markdown, thus reducing total sales. On the other hand, having a very large expected markdown encourages too many early consumers to wait, but some of them do not end up buying the product at markdown due to the emotional loss. This again hurts revenue. Hence, the retailer benefits from its consumers having an intermediate level of $\tilde{\Delta}$. We observe that this desirable level of $\tilde{\Delta}$ tends to increase with a larger inventory. This is because with sufficient

Figure 4 **Impact of Consumers' Expected Markdown on the Retailer's Optimal Markdown and Payoff**



(a) Optimal Markdown Δ^* (b) Optimal Payoff Π^*

Figure 5 **Retailer's Optimal Payoff versus the Difference between Optimal and Expected Markdown $\left|\Delta^* - \tilde{\Delta}\right|$**



(a) Low Inventory (b) High Inventory

Note. The x axis in both figures presents four cases of the absolute difference between the optimal and expected markdowns (in % points): (i) $\left|\Delta^* - \tilde{\Delta}\right| = 0$, (ii) $\left|\Delta^* - \tilde{\Delta}\right| \in (0,5]$, (iii) $\left|\Delta^* - \tilde{\Delta}\right| \in (5,30]$, and (iv) $\left|\Delta^* - \tilde{\Delta}\right| > 30$.

inventory, keeping a large number of consumers in the market (e.g., due to a large expected markdown) is beneficial. When instead the inventory is tight, the retailer can afford to have a smaller consumer base to ensure a good margin (with a smaller markdown) while mitigating potential underage cost.

Taking the above analysis further, Figure 5 demonstrates how the distribution of the retailer's optimal payoff (across all numerical instances) changes as the difference between the optimal markdown and the consumers' expected markdown, $\left|\Delta^* - \tilde{\Delta}\right|$, increases. We observe that the retailer in general earns a higher optimal payoff when Δ^* is equal or close to $\tilde{\Delta}$, especially when the retailer's inventory is large (see Figure 5b). That is, the retailer can benefit by making consumers' expected markdown aligned with its optimal markdown level. One way to do so is to pre-announce and commit to its markdown decision. Prior studies have shown that committing to a markdown schedule typically outperforms contingent pricing when consumers are forward-looking and fully rational (e.g., Aviv and Pazgal 2008). Our results indicate a behavioral reason for the benefit of commitment; i.e., it eliminates undesirable effects due to consumers forming an overly optimistic expected markdown. We also highlight that not all behavioral regularities of consumers can be leveraged by the retailer to its own benefit. Sometimes the retailer would be better off helping consumers avoid their own biases.

4. Quantifying the Impacts of Quality Perception and Loss Emotion

We next analyze the operational and financial impacts of consumers' quality perception and potential loss emotion (due to an optimistic expectation of future markdown). In particular, we investigate how much the retailer would lose if it made incorrect assumptions on some or both of these behavioral factors. The three scenarios we study are as follows:

(i) The retailer assumes that consumers do not experience any loss emotion after observing the actual markdown, i.e., $\eta = 0$, but that consumers form price-based quality perception.

(ii) The retailer assumes that all consumers have a fixed, exogenous quality perception that is not affected by the price, i.e., $q_1 = q_2 = Q_{\text{fixed}}$, but that consumers are subject to potential loss emotion. We choose $Q_{\text{fixed}} \in \{0.5p, 0.75p, p, 1.25p, 1.5p\}$.

(iii) The retailer assumes that consumers do not experience any loss emotion after observing the actual markdown *and* that all consumers have a fixed, exogenous quality perception, i.e., $\eta = 0$ and $q_1 = q_2 = Q_{\text{fixed}}$.

We employ the following procedure in our analysis. For each numerical instance, we first compute the retailer's optimal markdown and payoff given our model. We then compute the retailer's suboptimal markdown decision for each scenario of incorrect assumptions discussed above. Given the suboptimal markdown decision, we then re-evaluate consumers' purchase behavior under the correct parameters and compute the resulting payoff for the retailer in each scenario. Finally, we compare the retailer's suboptimal decision and the resulting payoff to the optimal ones. Table 1 summarizes our comparison results. Figure 6 presents how the suboptimal decisions compare to the optimal decisions across all numerical instances and the three scenarios we examine.

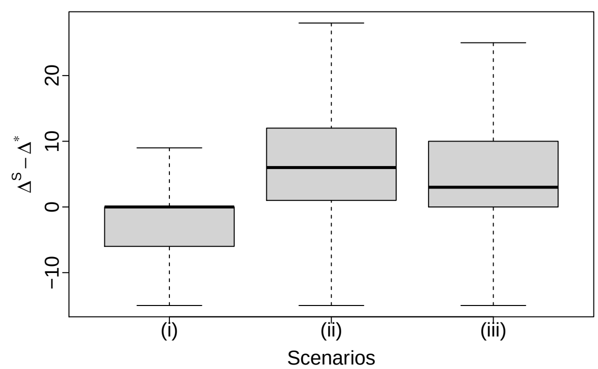
We highlight two key observations. First, the retailer on average errs on under-markdown (i.e., setting a markdown smaller than the optimal one) for scenario (i), when it mistakenly assumes $\eta = 0$. In particular, the retailer keeps its markdown small to prevent late consumers from having a low perceived quality; however, it overlooks the negative effect of (some) returning consumers experiencing emotional loss and eventually not buying the product. Conversely, the retailer tends to set a markdown that is too large (compared to the optimal decision) when it mistakenly assumes that consumers' quality perception is fixed and does not depend on price-related information. This is mainly because the retailer ignores the negative effect that a markdown has on returning and late consumers' quality perception. Second, we observe from the payoff losses summarized in Table 1 that ignoring consumers' price-based

Table 1 Suboptimal Decision and Payoff Loss When Ignoring Quality Perception and Loss Emotion

	Scenarios of incorrect assumptions					
	Low inventory			High inventory		
	(i) $\eta=0$	(ii) $q_1=q_2=Q_{\text{fixed}}$	(iii) $\eta=0$ and $q_1=q_2=Q_{\text{fixed}}$	(i) $\eta=0$	(ii) $q_1=q_2=Q_{\text{fixed}}$	(iii) $\eta=0$ and $q_1=q_2=Q_{\text{fixed}}$
Mean $\frac{\Delta^S}{p}-\frac{\Delta^*}{p}$ (% off)	−3.62	7.96	4.96	−4.76	6.23	3.77
Std. dev. $\frac{\Delta^S}{p}-\frac{\Delta^*}{p}$ (% off)	6.42	10.42	11.50	8.34	10.19	11.44
Mean $\left\lvert\frac{\Delta^S}{p}-\frac{\Delta^*}{p}\right\rvert$ (% off)	3.62	9.48	8.20	4.79	7.91	7.99
Std. dev. $\left\lvert\frac{\Delta^S}{p}-\frac{\Delta^*}{p}\right\rvert$ (% off)	6.42	9.07	9.47	8.33	8.95	9.01
Mean $\frac{\Pi^*-\Pi^S}{\Pi^*}\times 100\%$	9.87	38.61	29.06	3.93	6.30	6.13
Std. dev. $\frac{\Pi^*-\Pi^S}{\Pi^*}\times 100\%$	20.36	44.81	40.38	9.38	15.42	15.42
Worst-case $\frac{\Pi^*-\Pi^S}{\Pi^*}\times 100\%$	100.00	486.99†	486.99†	100.00	100.00	100.00
Fraction of instances $\Delta^S\neq\Delta^*$ (%)	42.33	83.31	77.69	43.52	82.94	83.23

Notes. The variables Δ^S and Π^S denote the retailer's suboptimal markdown and suboptimal payoff under incorrect assumptions of consumers' behavior. "Std. dev." stands for standard deviation. † The worst case loss can be larger than 100% because suboptimal decisions can lead to a negative payoff ($\Pi^S<0$) due to the inventory underage cost.

Figure 6 Distribution of $(\Delta^S-\Delta^*)$ (in %) across Scenarios with Incorrect Assumptions of Consumer Behavior



Note. The three scenarios are: (i) $\eta=0$; (ii) $q_1=q_2=Q_{\text{fixed}}$; (iii) $\eta=0$ and $q_1=q_2=Q_{\text{fixed}}$.

quality perception has a more substantial adverse impact on the retailer's payoff when inventory is tight. In contrast, when inventory is sufficient, ignoring consumers' potential loss emotion has a more substantial adverse impact on the retailer's payoff. That is, retailers with scarce inventory should make conscious efforts to correctly understand how their markdown pricing could influence the consumers' quality perception. Conversely, retailers with ample inventory should be particularly mindful of the potential loss emotion that their markdowns could create among their consumers.[3]

[3] In addition to the analyses presented, we numerically analyze a number of model extensions as robustness checks: (i) when early consumers form rational expectation of product availability in Period 2; (ii) when the retailer incurs a (sunk) inventory cost; (iii) when early and late consumers have different valuations on (perceived) quality relative to price (i.e., $\theta_{\max}$ is different for early vs. late consumers); (iv) when returning consumers do not experience loss emotions. We confirm that our main insights remain unchanged in these extensions.

5. The Consumer Study

To provide further empirical grounding of the consumer model developed, we design and implement an online consumer study in which participants face a hypothetical situation of shopping for a new dress shirt (for male participants) or a new blouse (for female participants). The study is designed to answer the following questions: (i) How does price-related information affect early and late consumers' quality perception of the product? (ii) What functional relationships reasonably characterize the effects of price-related information on early and late consumers' quality perception? (iii) Does the discrepancy between consumers' expected markdown and the actual markdown create any gain or loss emotions that affect their purchase decisions?

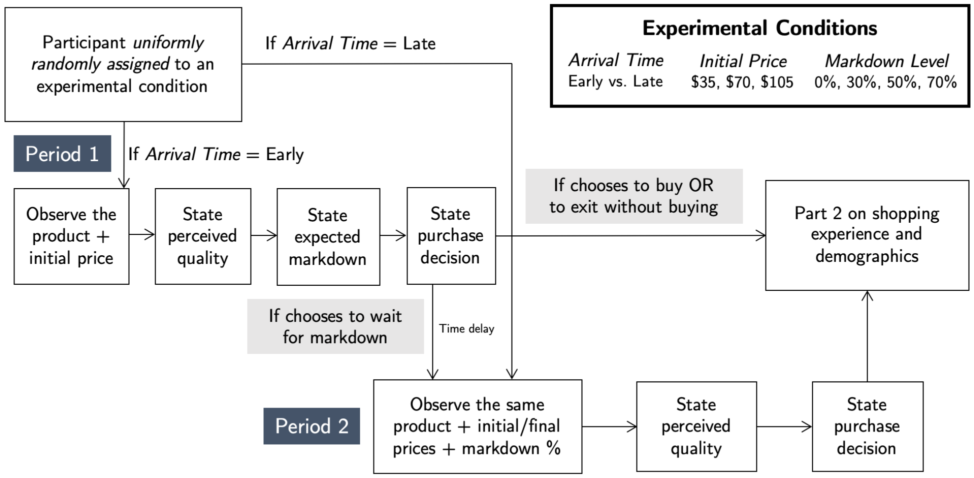
5.1. Study Design and Procedure

Study design: The key treatment variables we manipulate in the study are a participant's arrival time to the store, the initial price of the product in Period 1, and the level of markdown applied to the product in Period 2. We employ a 2 (arrival time: early vs. late) $\times$ 3 (initial price: \$35, \$70, \$105) $\times$ 4 (markdown level: 0%, 30%, 50%, 70%) between-subject factorial design. That is, each participant only experiences one of the twenty-four treatment conditions. We randomly assign participants to these treatment conditions in a balanced fashion so that each condition involves 20 participants. We include a 0% markdown to capture the scenario where the product is not marked down in Period 2. Participants assigned to the early arrival time condition (early consumers) arrive in Period 1 when the product is sold at its initial price. They are informed that the product may be marked down at a later time. Conversely, participants assigned to the late arrival time condition (late consumers) arrive in Period 2 when the product is already marked down (if any). The gap between these two periods is fixed at three months and explicitly shown to the participants. To solely focus on the effect of price information on consumers' quality perception, we assure early consumers that the product will be available in Period 2 should they decide to wait for a markdown. Thus, the only friction against waiting for a markdown in our study is the time spent waiting for Period 2 to come (simulated by a delay in the study). In §5.3, we confirm the validity of our results in a robustness study where a stockout is possible in Period 2.

Participants and procedure: We conducted the study on Amazon Mechanical Turk. In total, 958 participants completed the study. 50% of them identified themselves as male; the median age was 33 years old with a standard deviation of 11 years. The study consists of two parts. In the first part, participants evaluate the product shown, state their quality perception and expected markdown, and make purchase decisions. In the second part, they answer questions regarding their general experience of buying similar products and their demographics. We next explain in more detail the general flow that a participant goes through during the study, as illustrated in Figure 7.

If a participant is assigned to be an early consumer, then the participant is shown a picture of the product along with some descriptions of the product's characteristics, including its initial price at which

Figure 7 Treatment Conditions and Study Flow



the product is currently sold, and the fact that it may be marked down to a lower price in three months. The participant is asked to state their quality perception of the product on a 0–100 scale. A higher value means the product is perceived to have higher quality. The participant also answers how much percentage discount they expect to be applied to the product in three months. Afterwards, the participant chooses one of three options: (a) buying the product now at its initial price, (b) waiting for a markdown and returning in Period 2, or (c) leaving the store without buying. If the participant chooses (a) or (c), then they will be directed to the second part of the study. If instead, the participant chooses (b), then they will be directed to Period 2 and will observe the exact same product with markdown information, including the initial price, the percentage discount off, and the final selling price. Given the new information, the participant is again asked to state their quality perception of the product. Finally, the participant indicates whether to buy the product or leave the store without buying, after which they are directed to the second part of the study. If a participant is assigned to be a late consumer, then the participant is directed to Period 2 immediately. This means that the participant observes the product, its characteristics, and the markdown information as discussed above. The participant is also told that the product was sold at its initial price three months ago. They then state their quality perception of the product on a 0–100 scale and indicates their purchase decision: buy the product or leave without buying. The participant is finally directed to the second part of the study.

In the second part of the study, all participants answer a series of questions regarding their general experience of shopping for similar products as well as provide demographic information including gender, age, income level, and highest education. Participants received a flat rate compensation of $2.00 for completing our study, which took on average 7.05 minutes. We follow best practices and established protocols for perceptional and judgmental tasks (e.g., Tversky and Shafir 1992, Gal and Rucker 2011, Bitterly et al. 2017) and for online studies using the Amazon Mechanical Turk platform (e.g., Paolacci

Table 2 Summary of Participants' Purchase Decisions

	Buy	Exit	Return-Buy	Return-Exit	Total
Early consumers	24	179	190	82	475
Late consumers	189	294	–	–	483
Total	213	473	190	82	958

et al. 2010, Buhrmester et al. 2011, Mason and Suri 2012). Further details are provided in Supplementary Appendix E.

5.2. Study Results

Table 2 presents a summary breakdown of the participants' purchase decisions. The categories Return-Buy and Return-Exit correspond to early consumers who choose to wait for a markdown and eventually purchase the product or exit the market without buying.

5.2.1. Consumers' quality perception in Period 1. A total of 475 participants (49.6%) are assigned to be early consumers; i.e., they arrive in Period 1. To analyze the relationship among early consumers' quality perception, the product's initial price, and the consumers' expected markdown, we estimate a series of regression models with the participants' quality perception as the dependent variable and the initial price and the expected markdown as key independent variables (see Appendix C.1, Table C1). We observe a significantly positive coefficient for the initial price and a significantly negative coefficient for the expected markdown. Hence, our data confirms that *consumers' quality perception in Period 1 increases with the product's initial price but it is negatively correlated with their expected markdown.*

5.2.2. Consumers' quality perception in Period 2. A total of 483 participants (50.4% of participants) are assigned to arrive in the second period, while 272 participants (57.26% of early consumers) choose to wait for the markdown. We estimate a set of regression models in which the dependent variable is the quality perception of these consumers and the key independent variables are the product's initial price or selling price and the markdown applied (see Appendix C.2, Table C2). The regression estimates for the initial/selling price are significantly positive while the estimate for the markdown is significantly negative. Therefore, our data confirms that *consumers' quality perception in Period 2 increases with the product's initial price but decreases with the markdown applied; taken together, it increases with the product's final selling price.*

5.2.3. Functional relationships between consumers' quality perception and price-related information. Here we use the data from the consumer study to validate the specific functional forms of price-based quality perception used in our consumer model in §3.1. To do so, we first use stepwise model selection to identify the best polynomial models that describe both early and late consumers' price-based quality perception. We then compare these best polynomial models to the simple linear models assumed in Equations (1) and (2) with respect to both in-sample fit and out-of-sample prediction. The detailed procedure is discussed in Appendix C.3.1 and the results are presented in Table C3. The results confirm that the simple linear models achieve comparably good in-sample and out-of-sample performance and hence, can be considered as reasonable characterization of consumers' price-based quality perception.

5.2.4. Gain/loss emotions from mismatch between expected and actual markdown. A final behavioral validation we address is whether returning consumers experience any gain or loss emotions when the actual markdown applied is different from their expectation. We generalize Equation (4) in §3.1 to also allow for potential gain emotions when returning consumers observe a larger-than-expected markdown. In particular, returning consumers' utility of purchasing the product at the markdown price $(p-\Delta)$ is modeled as $U_{\text{Buy}} = \theta Q - (p-\Delta) + \Psi(\Delta - \tilde{\Delta})$, where $\Psi(\Delta - \tilde{\Delta}) = \eta_1(\Delta - \tilde{\Delta})$ if $\Delta - \tilde{\Delta} \geq 0$ and $\Psi(\Delta - \tilde{\Delta}) = \eta_2(\Delta - \tilde{\Delta})$ if $\Delta - \tilde{\Delta} < 0$. The parameter θ is assumed to be uniformly distributed on $[a,b]$ and Q is the participant's stated perceived quality. The term $\theta Q - (p-\Delta)$ captures the consumption utility from the purchase, while $\Psi(\Delta - \tilde{\Delta})$ captures the potential gain/loss utility. The notion of loss aversion predicts that $\eta_2 > \eta_1 \geq 0$. Using the data of returning consumers in our consumer study, we structurally estimate the parameters a, b, η_1, and η_2 as well as the following nested models: (i) $\eta_1 = 0$ (consumers only experience losses), (ii) $\eta_1 = \eta_2 \geq 0$ (gains and losses have the same weight), and (iii) $\eta_1 = \eta_2 = 0$ (consumers do not experience any gains or losses). The estimation results are presented in Appendix C.4. The likelihood ratio tests reject models (ii) and (iii) in favor of the full model and model (i), while model (i) cannot be rejected. Hence, our analysis confirms that returning consumers exhibit significant loss emotion when the actual markdown is lower than expected but do not exhibit significant gain emotion when observing a larger-than-expected markdown.[4]

5.3. Robustness Study: Product Availability and Perceived Quality

To examine whether our main results related to early consumers might be affected by product availability, we design and implement a second study that focuses on early consumers and allows for a possible stockout in Period 2. In particular, the treatment conditions follow a 2 (initial price: \$70 vs. \$105) × 2 (availability: 50% vs. 100%) × 3 (markdown level: 0, 30%, or 70%) design. The design and flow of the study remain the same as in our main study except for the following changes. For product availability, the participants observe on the product page either one of the following statements depending on the condition they are in: (i) "There is a 50% chance that it [the product] would be available in six weeks";

[4] We have conducted additional analysis to estimate the model parameters based on the data from all participants in our study and validate the effectiveness of our model in predicting consumers' purchase decisions out of sample. In particular, given the participants' reported quality perception, expected markdown, purchase decisions, and the price and markdown values in the study, we estimate $\theta_{\max}$, β, and η by adopting the simulated maximum likelihood approach. To assess our model's out-of-sample performance, we also estimate a benchmark model in which consumers have a fixed quality perception and do not experience loss emotions (only estimating $\theta_{\max}$ and β). Given these estimated parameter values, we simulate the purchase decisions of the participants in the testing set under both our model and the benchmark model and find that our model provides a significantly better out-of-sample likelihood.

or (ii) "It [the product] is guaranteed to be in stock in six weeks." Prior to Period 2, we randomly determine the product's availability for each returning consumer based on the availability condition they are in. If a stockout occurs, the participant will be informed that the product is no longer available and asked to state their perceived quality again. Otherwise, the markdown is revealed to the participant, and the participant needs to both state their perceived quality and make purchase decision. A total of 447 participants from Amazon Mechanical Turk who had not participated in our main study completed this study and passed our attention checks. Half of them are male, and their median age is 37 years old with a standard deviation of 12. Further details are discussed in Supplementary Appendix F.

We make three key observations from this study. First, given the same initial price, the quality perception stated by the participants in the new data is not significantly different from that in our original data, and the stated quality perception between the two availability conditions is not significantly different (Wilcoxon rank sum tests, $p > 0.1$). Second, re-estimating our regression models with the pooled data, we again observe early consumers' quality perception to be increasing in the product's initial price and decreasing in their expected markdown, consistent with our results in §5.2.1 (see Table F8). Third, following the model selection process in §5.2.3, we again confirm that the simple linear model reasonably characterizes early consumers' price-based quality perception regardless of product availability in Period 2 (see Table F9).

6. Conclusions

In this paper, we develop a bi-level optimization model to investigate how consumers' quality perception and expectation on the future markdown affect the retailer's optimal markdown pricing strategy. We consider a retailer selling a fixed inventory of a product over two periods where a markdown might be offered in the second period. A fraction of consumers arrive at the market in the first period (early consumers), while the rest of them arrive in the second period (late consumers). Consumers form quality perception given price-related information available to them when they arrive at the market.

Our consumer model captures both consumers' price-based quality perception and their potential loss emotion due to an optimistic expectation of the future markdown. The model is both motivated by prior empirical results in the literature and validated by an online consumer study we design and conducted. We embed the consumer model in a bi-level optimization framework to analyze the impact of consumers' behavioral factors as well as inventory on the retailer's optimal markdown strategy. We fully characterize consumers' purchase behavior under a markdown strategy and the conditions under which it is optimal for the retailer to apply a markdown. We demonstrate that the optimal markdown is non-monotonic in the consumers' quality perception parameters. Furthermore, our results show that it is in the retailer's best interest to pre-announce and commit to a markdown strategy to eliminate a mismatch between consumers' expectation about the markdown and the actual markdown applied.

In addition, we quantify that ignoring the behavioral factors captured in our consumer model can substantially hurt the retailer's payoff. When inventory is tight, ignoring consumers' price-based quality perception leads to 38% loss in payoff on average. When instead inventory is sufficient, the retailer should be particularly mindful of the potential emotional loss that its price markdown could create among its consumers.

Our paper is among a handful of recent studies in the revenue management field that begin to capture salient behavioral regularities in consumers' purchase decisions. Our model particularly incorporates the new dimension of price-based quality perception, in addition to the presence of loss emotion (when relaxing the rational expectations assumption on consumers). By developing richer and more realistic consumer models, we can improve the practicality of our models to generate effective and actionable recommendations. We hope that our paper can stimulate more research in this important direction.

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Appendix A: Proofs of Analytical Results

A.1. Proofs of Proposition 1 and Proposition 2

Proof of Proposition 1: We assume that $\forall a, t,$ and $\Delta \le p : q_0 + ap - t\Delta > 0$. We first prove part (i). Let U_i denote the consumers' utility of choosing an option i: $U_i = \theta q_i - p_i$, with $i \in \{0, 1, w\}$, where $U_0 = 0$ is the utility of leaving without buying, U_1 is the utility of buying the product at its initial price, and U_w is the utility of waiting to return in Period 2. Each consumer chooses an option that maximizes her utility. The consumer chooses option i if and only if $U_i \ge U_j$ for $j \ne i$. Recall from §3.1 that $U_1(\theta) = \theta q_1 - p$ and $U_w(\theta) = \beta(\theta q_1 - (p - \tilde{\Delta}))$. The consumer would buy the product at the initial price if and only if $U_1(\theta) \ge 0$ and $U_1(\theta) \ge U_w(\theta)$; i.e., $\theta \ge \bar{\theta} \equiv \frac{p}{q_1} + \frac{\tilde{\Delta}\beta}{q_1(1-\beta)}$. Let $\hat{\theta} = \min(\bar{\theta}, \theta_{max})$, then the consumer would buy the product at the initial price if and only if $\theta \in [\hat{\theta}, \theta_{max}]$. Similarly, the consumer would leave the store without buying if and only if $U_1(\theta) < 0$ and $U_w(\theta) < 0$; i.e., $\theta < \underline{\theta} \equiv \frac{p-\tilde{\Delta}}{q_1}$. Finally, the consumer would choose to wait and return in Period 2 if and only if $U_w(\theta) \ge 0$ and $U_w(\theta) > U_1(\theta)$; i.e., $\theta \in [\underline{\theta}, \hat{\theta})$. We in fact verify that $\underline{\theta} < \bar{\theta}$. We next prove part (ii). For those early consumers who choose to wait in Period 1, they return in Period 2 and choose between buying the product with a discount Δ applied and leaving without buying. Recall from §3.1 that the utility of buying the product in Period 2 is given by $U_2(\theta) = \theta q_2 - (p - \Delta) - \eta\left(\tilde{\Delta} - \Delta\right)^+$, where $x^+ = max(x, 0)$ and $q_2 = q_0 + a(p - \Delta)$. Returning consumers would purchase the product in Period 2 if and only if $\theta \in [\underline{\theta}, \hat{\theta})$ and $U_2(\theta) \ge 0$; i.e., $\theta \in [\max\{\underline{\theta}, \theta'(\Delta)\}, \hat{\theta})$, where $\theta'(\Delta) \equiv \frac{p}{q_2} - \frac{\Delta - \eta(\tilde{\Delta}-\Delta)^+}{q_2}$. If $\underline{\theta} \ge \theta'(\Delta)$, then all returning consumers buy the product in Period 2. Since θ' is decreasing in Δ, we have $\underline{\theta} \ge \theta'(\Delta)$ iff $\Delta \ge \Delta_1 = \max\{\frac{p+\eta\tilde{\Delta}-(q_0+ap)\underline{\theta}}{1+\eta-a\underline{\theta}}, \frac{p-(q_0+ap)\underline{\theta}}{1-a\underline{\theta}}\}$. In addition, some of the returning consumers would buy the product in Period 2 if $\theta'(\Delta) < \hat{\theta} \iff \Delta - \Delta_0 > 0$, where $\Delta_0 \equiv \frac{p+\eta\lambda\tilde{\Delta}-(q_0+ap)\hat{\theta}}{1+\eta-a\hat{\theta}}$, and we have $\Delta_0 \le \tilde{\Delta}$.

Let $\Delta_0^+ = \max\{0, \Delta_0\}$ and simplify $\theta'(\Delta)$ as θ'. Then: (i) If $\Delta \ge \Delta_1$, all returning consumers buy the product in Period 2. (ii) If $\Delta \in (\Delta_0^+, \Delta_1)$, then those returning consumers with $\theta \in [\theta', \hat{\theta})$ buy the product in Period 2, and those with $\theta \in [\underline{\theta}, \theta')$ leave the market without buying. (iii) If $\Delta \le \Delta_0^+$, then none of the returning consumers buy the product in Period 2.

Proof of Proposition 2: Late consumers arrive in Period 2 and can either buy the product or exit without purchasing. Their utility of buying the product is given by $U_2^2(\theta) = \theta q_2 - (p - \Delta)$. Hence, a late consumer buys the product if and only if $U_2^2(\theta) \ge 0$; i.e., $\theta \ge \tilde{\theta} \equiv \frac{p-\Delta}{q_2}$. □

A.2. Proof of Lemma 1

Based on Propositions 1 and 2, we have $D_1 = 1 - \frac{\hat{\theta}}{\theta_{max}}$, $D_3 = 1 - \frac{\tilde{\theta}}{\theta_{max}}$, and $D_2 = \frac{(\hat{\theta} - \max\{\underline{\theta}, \theta'\})^+}{\theta_{max}}$ and thus:

(i) $D_1 = \frac{1}{\theta_{max}}\left(\theta_{max} - \frac{p}{q_1} - \frac{\tilde{\Delta}\beta}{q_1(1-\beta)}\right)^+$ is decreasing in $\tilde{\Delta}$, and β and is independent of Δ.

(ii) $D_3 = \frac{1}{\theta_{max}}\left(\theta_{max} - \frac{p-\Delta}{q_0+a(p-\Delta)}\right)^+$ is increasing in Δ.

(iii) $D_2 = \begin{cases} \frac{1}{\theta_{max}}(\hat{\theta} - \underline{\theta}), & \text{if } \Delta \ge \Delta_1, \\ \frac{1}{\theta_{max}}(\hat{\theta} - \theta'(\Delta))^+, & \text{if } \Delta \le \Delta_1 \end{cases}$ and $\theta'(\Delta)$ is decreasing in Δ. Hence, D_2 is non-decreasing in Δ.

With respect to β, because $\hat{\theta}$ is non-decreasing in β, D_2 is non-decreasing in β.

Finally, with respect to $\tilde{\Delta}$, we can show that $\hat{\theta}$ is non-decreasing in $\tilde{\Delta}$ and $\underline{\theta}$ is decreasing in $\tilde{\Delta}$.

When $\bar{\theta} < \theta_{max}$ and $\underline{\theta} < \theta'$, then $D_2 = \frac{\bar{\theta} - \underline{\theta}}{\theta_{max}}$ and is non-decreasing in $\tilde{\Delta}$. Let $\bar{\Delta}$ and $\underline{\Delta}$ such that $\bar{\theta} = \theta_{max}$ at $\tilde{\Delta} = \bar{\Delta}$ and $\underline{\theta} = \theta'$ at $\tilde{\Delta} = \underline{\Delta}$. Thus if $\tilde{\Delta} < \min\{\underline{\Delta}, \bar{\Delta}\}$, then D_2 is non-decreasing in $\tilde{\Delta}$.

Suppose $\tilde{\Delta} \ge \min\{\underline{\Delta}, \bar{\Delta}\}$. We have two cases:

- *Case 1*: If $\bar{\Delta} \leq \underline{\Delta}$, then when $\tilde{\Delta} \in (\bar{\Delta}, \underline{\Delta})$, we have $D_2 = \frac{\theta_{max}-\theta}{\theta_{max}}$ and is non-decreasing in $\tilde{\Delta}$. When $\tilde{\Delta} \geq \underline{\Delta}$, $D_2 = \frac{(\theta_{max}-\theta')^+}{\theta_{max}}$ and is non-increasing $\tilde{\Delta}$. We conclude that D_2 is non-decreasing in $\tilde{\Delta}$ for $\tilde{\Delta} < \underline{\Delta}$ and is non-increasing in $\tilde{\Delta}$ for $\tilde{\Delta} \geq \underline{\Delta}$.
- *Case 2*: If $\bar{\Delta} < \underline{\Delta}$, then if $\tilde{\Delta} < \Delta$, we have $D_2 = \frac{(\bar{\theta} - \frac{p-\Delta}{q_2(\Delta)})^+}{\theta_{max}}$ and for $\tilde{\Delta} \geq \Delta$, $D_2 = \frac{(\bar{\theta}-\theta')^+}{\theta_{max}}$. Thus D_2 is non-decreasing in $\tilde{\Delta} < \min\{\underline{\Delta}, \Delta\}$. If $\tilde{\Delta} \in [\min\{\underline{\Delta}, \Delta\}, \underline{\Delta})$, then we show there exists Δ_2 such that D_2 is increasing in $\tilde{\Delta}$ iff $\tilde{\Delta} < \Delta_2$. Thus if $\tilde{\Delta} \in [\min\{\underline{\Delta}, \Delta\}, \min\{\underline{\Delta}, \Delta_2\})$, then D_2 is increasing in $\tilde{\Delta}$. Otherwise, if $\tilde{\Delta} > \min\{\underline{\Delta}, \Delta_2\}$ then $D_2 = \frac{(\hat{\theta}-\theta')^+}{\theta_{max}}$ is non-decreasing in $\tilde{\Delta}$.

Denote $\tilde{\Delta}_1 \equiv max(\min\{\underline{\Delta}, \Delta_2\}, \min\{\underline{\Delta}, \Delta\})$, then we conclude that D_2 is non-decreasing over $[0, \tilde{\Delta}_1)$ and non-increasing over $(\tilde{\Delta}_1, p]$.

□

A.3. Proof of Theorem 1

First of all, let's assume that $\theta_{max} > \max\{\frac{p-\tilde{\Delta}}{q_1}, \frac{p-\tilde{\Delta}}{q_2(\tilde{\Delta})}\}$. This condition is equivalent to having at least a portion of early customers who will either buy the product at full price or at a discounted price. Otherwise, all early customers exit the market without buying, which is a trivial case.

The payoff function we consider can be written as follows:

$$\Pi(\Delta) = \gamma p D_1 + \gamma(p-\Delta)D_2(\Delta) + (1-\gamma)(p-\Delta)D_3(\Delta) - s[\gamma D_1 + \gamma D_2(\Delta) + (1-\gamma)D_3(\Delta) - C]^+.$$

Let $D_T(\Delta) = \gamma D_1 + \gamma D_2(\Delta) + (1-\gamma)D_3(\Delta)$. We know from Lemma 1 that $D_T(\Delta)$ is increasing in Δ, and let $\hat{\Delta} = D_T^{-1}(C)$. Π is continuous and differentiable almost everywhere. The derivate function Π' is discontinuous particularly in Δ_0^+, Δ_1, $\tilde{\Delta}$ (because of the discontinuity in Δ_1) and $\hat{\Delta}$ and we have $\Delta_0^+ < \min\{\Delta_1, \tilde{\Delta}\}$. We have shown that $D_2(\Delta) = 0$ for $\Delta \in [0, \Delta_0^+]$. Hence, we can write for $\Delta \in [0,p] - \{\Delta_0^+, \hat{\Delta}, \Delta_1, \tilde{\Delta}\}$ that $D_3(\Delta) = \frac{1}{\theta_{max}}\left(\theta_{max} - \frac{p-\Delta}{q_0+a(p-\Delta)}\right)^+$ and $D_2 = \begin{cases} \frac{1}{\theta_{max} q_1}\min\{\frac{\tilde{\Delta}}{(1-\beta)}, \theta_{max}q_1 - p + \tilde{\Delta}\}, & \text{if } \Delta \geq \Delta_1, \\ \frac{(\hat{\theta}-\theta'(\Delta))^+}{\theta_{max}}, & \text{if } \Delta \leq \Delta_1. \end{cases}$ and thus we can write :

$$\Pi''(\Delta) = \begin{cases} \frac{-2(1-\gamma)}{\theta_{max}}\frac{q_0^2}{(q_0+a(p-\Delta))^3} & \text{if } \Delta < \hat{\Delta}, \Delta \in [0, \Delta_0^+) \cup (\Delta_1, p] \\ \frac{-2(1-\gamma)}{\theta_{max}}\frac{q_0^2}{(q_0+a(p-\Delta))^3} - s\frac{2(1-\gamma)}{\theta_{max}}\frac{aq_0}{(q_0+a(p-\Delta))^3} & \text{if } \Delta > \hat{\Delta}, \Delta \in [0, \Delta_0^+) \cup (\Delta_1, p] \\ -2\gamma\frac{q_0(q_0+\eta(q_0+a(p-\tilde{\Delta})))}{\theta_{max}(q_0+a(p-\Delta))^3} - \frac{2(1-\gamma)}{\theta_{max}}\frac{q_0^2}{(q_0+a(p-\Delta))^3} & \text{if } \Delta \in (\Delta_0^+, \min\{\Delta_1, \tilde{\Delta}\}), \Delta < \hat{\Delta} \\ -2\frac{q_0^2}{\theta_{max}(q_0+a(p-\Delta))^3} & \text{if } \Delta \in (\Delta_0^+, \Delta_1), \tilde{\Delta} \leq \Delta < \hat{\Delta} \\ -2\gamma\frac{q_0(q_0+\eta(q_0+a(p-\tilde{\Delta})))}{\theta_{max}(q_0+a(p-\Delta))^3} - \frac{2(1-\gamma)}{\theta_{max}}\frac{q_0^2}{(q_0+a(p-\Delta))^3} & \\ -s\frac{2(1-\gamma)}{\theta_{max}}\frac{aq_0}{(q_0+a(p-\Delta))^3} - s\frac{2\gamma}{\theta_{max}}\frac{a(q_0+\eta(q_0+a(p-\tilde{\Delta})}{(q_0+a(p-\Delta))^3} & \text{if } \Delta \in (\Delta_0^+, \min\{\Delta_1, \tilde{\Delta}\}), \Delta > \hat{\Delta} \\ \frac{-2q_0^2}{\theta_{max}(q_0+a(p-\Delta))^3} - s\frac{2aq_0}{\theta_{max}(q_0+a(p-\Delta))^3} & \text{if } \Delta \in (\Delta_0^+, \Delta_1), \Delta > \max\{\hat{\Delta}, \tilde{\Delta}\} \end{cases}$$

We have $\Pi''(\Delta) < 0$ for $\Delta \notin \{\Delta_0^+, \hat{\Delta}, \Delta_1, \tilde{\Delta}\}$. Thus, Π is strictly concave over $[0, \Delta_0^+)$, $(\Delta_0^+, min(\Delta_1, \tilde{\Delta}))$, $(min(\Delta_1, \tilde{\Delta}), max(\Delta_1, \tilde{\Delta}))$ and $(max(\Delta_1, \tilde{\Delta}), p)$.

In particular, we have:

$$\frac{\partial \Pi}{\partial \Delta}(\Delta) = \begin{cases} (p-\Delta)\cdot(\gamma\frac{\partial D_2}{\partial \Delta}(\Delta) + (1-\gamma)\frac{\partial D_3}{\partial \Delta}(\Delta)) - (\gamma D_2(\Delta) + (1-\gamma)D_3(\Delta)) & \Delta \leq \hat{\Delta} \\ (p-s-\Delta)\cdot(\gamma\frac{\partial D_2}{\partial \Delta}(\Delta) + (1-\gamma)\frac{\partial D_3}{\partial \Delta}(\Delta)) - (\gamma D_2(\Delta) + (1-\gamma)D_3(\Delta)) & \Delta \geq \hat{\Delta} \end{cases}$$

Since D_2 and D_3 are non-decreasing in Δ, we conclude that $\frac{\partial \Pi}{\partial \Delta}(\hat{\Delta}^-) \leq \frac{\partial \Pi}{\partial \Delta}(\hat{\Delta}^+)$. Hence Π is strictly concave in $\hat{\Delta}$. Furthermore, since $s \geq p$, then $\frac{\partial \Pi}{\partial \Delta}(\Delta) \leq 0$ for $\Delta \geq \hat{\Delta}$ and Π is decreasing in Δ on $[\hat{\Delta}, p]$ If $\hat{\Delta} > \Delta_1$, then we have $\Pi'^-(\Delta_1) - \Pi'^+(\Delta_1) = \begin{cases} \gamma\frac{q_0+\eta(q_0+a(p-\tilde{\Delta}))}{\theta_{max}(q_0+a(p-\Delta_1))^2}(p-\Delta_1) & if\ \Delta_1 \leq \tilde{\Delta} \\ \gamma\frac{q_0}{\theta_{max}(q_0+a(p-\Delta_1))^2}(p-\Delta_1) & if\ \Delta_1 > \tilde{\Delta} \end{cases}$ and thus $\Pi'^-(\Delta_1) - \Pi'^+(\Delta_1) > 0$. This implies that in this case the payoff Π is strictly concave in Δ_1. If $\hat{\Delta} \leq \Delta_1$, then Π is strictly concave on $(\Delta_0^+, \hat{\Delta})$ and is decreasing on $[\hat{\Delta}, p]$.

Similarly, if $\hat{\Delta} > \tilde{\Delta}$, we have $\Pi'^{-}(\tilde{\Delta}) - \Pi'^{+}(\tilde{\Delta}) = \begin{cases} \frac{\gamma\eta}{\theta_{max}(q_0+a(p-\Delta_1))}(p-\tilde{\Delta}) & if\ \Delta_1 > \tilde{\Delta} \\ 0 & if\ \Delta_1 \leq \tilde{\Delta} \end{cases}$ and thus $\Pi'^{-}(\tilde{\Delta}) - \Pi'^{+}(\tilde{\Delta}) > 0$. If $\hat{\Delta} \leq \tilde{\Delta}$, Π is decreasing on $[\tilde{\Delta}, p]$. Hence, we conclude that Π is strictly concave over $(\Delta_0^+, \hat{\Delta})$ and is decreasing on $[\hat{\Delta}, p]$ and that Π has a unique optimum over (Δ_0^+, p).

Notice that if $\theta_{max} < \theta_{max}^1 = \frac{p+\eta\tilde{\Delta}}{q_0+ap}$, then $\Delta_0^+ > 0$ always. Hence, the payoff function Π is bimodal with at most two local maxima: $\Delta_1^* \in [0, \Delta_0^+]$ and $\Delta_2^* \in [\Delta_0^+, p]$. Otherwise, If $\Delta_0^+ = 0$, the payoff function is strictly concave and there is a unique optimum.

Case 1: If $\theta_{max} < \theta_{max}^1 = \frac{p+\eta\tilde{\Delta}}{q_0+ap}$, then $\Delta_0^+ > 0$ and the retailer offers markdown if and only if (i) $\Delta_1^* > 0$ **or** (ii) $\Pi(0) < \Pi(\Delta_2^*)$ are satisfied. In what follows, let $z = q_0 + ap$. We know that $\frac{p}{z} < min(\frac{p+\eta\tilde{\Delta}}{z}, \frac{p}{q_1} + \frac{\beta\tilde{\Delta}}{(1-\beta)q_1})$ for all $0 \leq \beta < 1$. Note then that $D_3(\Delta) = 0$ if and only if $\Delta \leq \Delta_3 = \frac{p-\theta_{max}z}{1-a\theta_{max}}$ and $D_2(\Delta) = 0$ if and only if $\Delta \leq \Delta_0$. Since $\theta_{max} > \frac{p-\tilde{\Delta}}{q_0+a(p-\tilde{\Delta})}$, we have $\Delta_3 < \Delta_0$ and if $\theta_{max} < \theta_{max}^0 = \frac{p}{z}$, we have $\Delta_3 > 0$. Thus, $\Pi(\Delta) = 0$ for $\Delta \in [0, \Delta_3]$, and in addition $\Pi'^{+}(\Delta_3) = (1-\gamma)(p-\Delta_3)D_3'^{+}(\Delta_3) > 0$. We conclude that if $\theta_{max} \leq \theta_{max}^0$, then $\Delta^* > 0$ always. If $\theta_{max} \in [\theta_{max}^0, \theta_{max}^1)$, then $D_3(\Delta) > 0$ for all $\Delta \in [0, p]$. We have $D_T(0) = \gamma D_1 + (1-\gamma)D_3(0) = \gamma(1 - \frac{\bar{\theta}}{\theta_{max}})^+ + (1-\gamma)(1 - \frac{\theta_{max}^0}{\theta_{max}})$. Let $\hat{C} = D_T(0)$. Consider each of the conditions separately:

(i) Π is strictly concave over $[0, \Delta_0^+)$ and thus:

$$\Delta_1^* > 0 \iff \Pi'(0) > 0 \iff \begin{cases} (1-\gamma)\frac{1}{\theta_{max}}\left[-\theta_{max} + \frac{2p}{z} - a\left(\frac{p}{z}\right)^2\right] > 0 & \text{if } \hat{\Delta} > 0 \\ (1-\gamma)\frac{1}{\theta_{max}}\left[-\theta_{max} + \frac{2p}{z} - a\left(\frac{p}{z}\right)^2 - s\frac{q_0}{z^2}\right]) > 0 & \text{if } \hat{\Delta} \leq 0 \end{cases}$$

$$\iff \begin{cases} \theta_{max} < \frac{p}{z} + \frac{pq_0}{z^2} & \text{if } C > \hat{C} \\ \theta_{max} < \frac{p}{z} + \frac{(p-s)q_0}{z^2} & \text{if } C \leq \hat{C} \end{cases}$$

Since $s \geq p$, we conclude that if $\theta_{max} \leq \theta_{max}^0$, then *it is always optimal for the retailer to offer markdown in Period 2.*

Let $\theta_{max}^2 = \frac{p}{z} + \frac{pq_0}{z^2}$. If $\theta_{max} < \theta_{max}^2$, then the $\Delta_1^* > 0$ iff $C > \hat{C}$. Otherwise, if $\theta_{max} \in [\theta_{max}^2, \theta_{max}^1)$, then condition (i) is never true. For the retailer to markdown in Period (2), condition (ii) has to be true. One can verify that $\Pi'(0) > \Pi'^{+}(\Delta_1)$. Since $\Pi'(0) \leq 0$, then $\Pi'^{+}(\Delta_1) < 0$ and Π is decreasing over $[\Delta_1, p]$. Since Π is also decreasing over $[\hat{\Delta}, p]$, this implies that $\Delta_2^* \in [\Delta_0^+, \min(\Delta_1, \hat{\Delta})]$.

(ii) It states that $\Pi(0) < \Pi(\Delta_2^*)$. One necessary condition for condition (ii) to hold if condition (i) is not held is that $\Pi'^{+}(\Delta_0) > 0$. This would insure that the payoff function is NOT decreasing over $[0, p]$.

We have $s \geq p$ and $\Delta_2^* \in [\Delta_0^+, \min(\Delta_1, \hat{\Delta})]$. In fact, because Π is concave over $[\Delta_0^+, \min(\Delta_1, \hat{\Delta})]$, we have: $\Delta_2^* = \max\{\min\{\Delta_{opt}^2, \hat{\Delta}, \Delta_1\}, \Delta_0^+\}$, where Δ_{opt}^2 verifies $\bar{\Pi}'(\Delta_{opt}^2) = 0$ and $\bar{\Pi}(\Delta) = \gamma pD_1 + (p-\Delta)(\gamma D_2(\Delta) + (1-\gamma)D_3(\Delta))$. One can show that $\Pi'^{+}(\Delta_0(\theta_{max}))$ is increasing in θ_{max}, and that there exists θ_{max}^3 such that $\Pi'^{+}(\Delta_0(\theta_{max})) > 0$ is equivalent to $\theta_{max} < \theta_{max}^3$. If $\bar{\Pi}(\Delta_{opt}^2) < \bar{\Pi}(0)$, then the retailer does not markdown, regardless of the value of capacity C. Otherwise, let $\Delta_4 \in (\Delta_0^+, \Delta_1]$ such that $\bar{\Pi}(\Delta_4) = \bar{\Pi}(0) = \gamma pD_1 + (1-\gamma)pD_3(0)$. Then, it is optimal for the retailer to markdown if $\hat{\Delta} \geq \Delta_4$. Hence, condition (ii) could be written as follows:

$$\Pi(0) < \Pi(\Delta_2^*) \iff \begin{cases} \gamma pD_1 + (1-\gamma)pD_3(0) < \bar{\Pi}(\Delta_{opt}^2) \\ \hat{\Delta} \geq \bar{\Pi}^{-1}(\gamma pD_1 + (1-\gamma)pD_3(0)) \end{cases}$$

$$\iff \begin{cases} (p-\Delta_{opt}^2)((1-\gamma)D_3(\Delta_{opt}^2) + \gamma D_2(\Delta_{opt}^2)) > (1-\gamma)pD_3(0) \\ C \geq D_T(\bar{\Pi}^{-1}(\gamma pD_1 + (1-\gamma)pD_3(0))) \end{cases}$$

Since $\bar{\Pi}'(\Delta^2_{opt}) = 0$, we can characterize both Δ^2_{opt} and $\bar{\Pi}(\Delta^2_{opt})$ analytically. Let $f(\theta_{max}) = \bar{\Pi}(\Delta^2_{opt}) - (1-\gamma)pD_3(0) - \gamma pD_1$. Furthermore, we can show that f as a function of θ_{max} has a unique zero θ^4_{max} such that $f(\theta_{max}) > 0$ iff $\theta_{max} < \theta^4_{max}$. We have shown that *for* $\theta_{max} \in (\theta^2_{max}, \min(\theta^1_{max}, \max\{\theta^3_{max}, \theta^4_{max}\}))$, *the retailer applies markdown, if and only if* $C > C_1 = D_T(\bar{\Pi}^{-1}(\gamma pD_1 + (1-\gamma)pD_3(0)))$. *If* $\theta^1_{max} > \max\{\theta^3_{max}, \theta^4_{max}\}$, *then the retailer never applies markdown when* $\theta_{max} \geq \max\{\theta^3_{max}, \theta^4_{max}\}$.

Case 2: For $\theta_{max} \in (\theta^1_{max}, \max\{\theta^3_{max}, \theta^4_{max}\})$, $\Delta^+_0 = 0$, and the payoff function is concave over $[0, p]$, a markdown that is strictly positive is possible only if $\Pi^{+'}(0) > 0$ and $\hat{\Delta} > 0$. This is equivalent to $\theta_{max} < \theta^5_{max} = \theta^2_{max} + \gamma pD'_3(0)$ and $C \geq D_T(0)$. We conclude that *if* $\theta_{max} \in (\theta^1_{max}, \theta^5_{max})$, *the retailer applies markdown iff* $C \geq \hat{C}$ *and if* $\theta_{max} \leq \theta^5_{max}$, *not applying markdown is optimal.*

Define the thresholds: $\underaccent{\bar}{C} = \max(\hat{C}, C_1)$, $\theta_0 = \theta^0_{max}$ and $\theta_1 = \max\{\theta^5_{max}, \min\{\theta^1_{max}, \max\{\theta^4_{max}, \theta^3_{max}\}\}\}$. We summarize the results as follows: (i) if $\theta_{max} < \theta_0$, then the retailer always applies price markdown. (ii) If $\theta_{max} \in [\theta_0, \theta_1)$, then the retailer applies markdown if and only if $C \geq \underaccent{\bar}{C}$. (iii) If $\theta_{max} \geq \theta_1$, then the retailer never applies markdown. □

A.4. Proof of Proposition 3

(i) From Theorem 1, we see that if $\theta_{\max} \geq \theta_2$, then the optimal markdown $\Delta^* = 0$ and is independent of inventory C.

(ii) If else, from Theorem 1, there exists a capacity threshold $\underaccent{\bar}{C}$ such that for $C \geq \underaccent{\bar}{C}$, the optimal markdown $\Delta^* > 0$. Recall that the payoff function has two local optimas $\Delta^*_1 \in [0, \Delta^+_0)$ and $\Delta^*_2 \in [\Delta^+_0, p]$. Let $\bar{\Pi}(\Delta) = \gamma pD_1 + (p - \Delta)(\gamma D_2(\Delta) + (1-\gamma)D_3(\Delta))$, Δ^1_{opt} and Δ^2_{opt} the discounts that verify the KKT conditions over $[0, \Delta^+_0]$ and $[\Delta^+_0, p]$ such that $\bar{\Pi}'(\Delta^1_{opt}) = 0$ and $\bar{\Pi}'(\Delta^2_{opt}) = 0$ (or $\Delta^2_{opt} = \Delta_1$ if there is no interior point that satisfies the KKT condition). Note that both Δ^1_{opt} and Δ^2_{opt} do not depend on capacity C. We have: $\frac{\partial \Pi}{\partial \Delta}(\Delta) = \begin{cases} (p-\Delta) \cdot (\gamma \frac{\partial D_2}{\partial \Delta}(\Delta) + (1-\gamma)\frac{\partial D_3}{\partial \Delta}(\Delta)) - (\gamma D_2(\Delta) + (1-\gamma)D_3(\Delta)) & \Delta \leq \hat{\Delta} \\ (p-s-\Delta) \cdot (\gamma \frac{\partial D_2}{\partial \Delta}(\Delta) + (1-\gamma)\frac{\partial D_3}{\partial \Delta}(\Delta)) - (\gamma D_2(\Delta) + (1-\gamma)D_3(\Delta)) & \Delta \geq \hat{\Delta} \end{cases}$

Since $s \geq p$, then $\frac{\partial \Pi}{\partial \Delta}(\Delta) \leq 0$. So the objective is decreasing in discount on $[\hat{\Delta}, p]$ and that means that optimal discount $\Delta^* \leq \hat{\Delta}$. Since $C > \underaccent{\bar}{C}$, we know at least that $\Delta^*_2 > \Delta^+_0$. Let $\underaccent{\bar}{C}_0 \leq \underaccent{\bar}{C}_1 < \underaccent{\bar}{C}_2$ be threshold capacities for which $\hat{\Delta} = 0$, $\hat{\Delta} = max(0, \Delta^1_{opt})$ and $\hat{\Delta} = \Delta^2_{opt}$ respectively. First notice that $\hat{\Delta}$ is increasing in C, so if $\hat{\Delta}(C) \leq 0$, i.e. $C \leq \underaccent{\bar}{C}_0$, then the objective function is decreasing over $[0, p]$ and the optimal discount is $\Delta^* = 0$. Note that in this case $\underaccent{\bar}{C}_0 = \underaccent{\bar}{C}$. If $C \in (\underaccent{\bar}{C}_0, \underaccent{\bar}{C}_1]$, then $\frac{\partial \Pi^-}{\partial \Delta}(\hat{\Delta}) > 0$ and we have that $\Delta^* = \Delta^*_1 = \hat{\Delta} = D_T^{-1}(0)$ and it is increasing with C. Since the payoff function is bimodal, we have that $\Delta^* = \Delta^*_1$, if $\Pi(\Delta^*_1) \geq \Pi(\Delta^*_2)$ and $\Delta^* = \Delta^*_2$, if $\Pi(\Delta^*_2) > \Pi(\Delta^*_1)$. In the case that $\Pi(\Delta^2_{opt}) \leq \Pi(\Delta^*_1)$, then for all $C > \underaccent{\bar}{C}_1$, $\Delta^* = \Delta^*_1 = max(0, \Delta^1_{opt})$ and it is independent of capacity C. Otherwise, if $\Pi(\Delta^2_{opt}) > \Pi(\Delta^*_1)$, there exists $\Delta_r \in (\Delta^+_0, p)$ such that $\bar{\Pi}(\Delta_r) = \bar{\Pi}(\Delta^*_1)$. Let $\bar{C}_r$ be such that $\hat{\Delta} > \Delta_r$ iff $C > \bar{C}_r$. Hence, if $C \in [\underaccent{\bar}{C}, \bar{C}_r)$, then $\Delta^* = \Delta^*_1$ and it is independent of capacity C. If $C \geq \bar{C}_r$, then $\Delta^* = min(\Delta^2_{opt}, \hat{\Delta})$ and is non-decreasing in C. We conclude that Δ^* *is non-decreasing in* C *for* $C \geq \underaccent{\bar}{C}$.

(iii) We have that $\Pi^* = max(\bar{\Pi}(\Delta^2_{opt}), \bar{\Pi}(\Delta^{1+}_{opt}), \Pi(\hat{\Delta}))$. $\bar{\Pi}(\Delta^2_{opt})$ and $\bar{\Pi}(\Delta^{1+}_{opt})$ are independent of C, and we have $\Pi(\hat{\Delta})$ is nondecreasing in C. We can conclude that Π^* is nondecreasing in C. □

A.5. Proof of Proposition 4

(i) We have $\Delta_2^* \in \{min(\Delta_{opt}^2, \hat{\Delta}), \Delta_1\}$. We also have that Δ_1 is increasing in a and $min(\Delta_{opt}^2, \hat{\Delta})$ is non-increasing in a (using Topkis' theorem). Furthermore, let $\bar{\Pi}(\Delta) = \gamma p D_1 + (p-\Delta)(\gamma D_2(\Delta) + (1-\gamma)D_3(\Delta))$. We have that $\Delta_2^* = min(\Delta_{opt}^2, \hat{\Delta})$ when $\bar{\Pi}'^{-}(\Delta_1) < 0$ or $\bar{\Pi}'^{+}(\Delta_1) > 0$ and $\Delta_2^* = \Delta_1$ when $\bar{\Pi}'^{-}(\Delta_1) > 0 > \bar{\Pi}'^{+}(\Delta_1)$. Equivalently, there exist $a_1 \le a_2$ such that $\Delta_2^* = min(\Delta_{opt}^2, \hat{\Delta})$ if $a < a_1$ or $a \ge \tilde{a}_2$, otherwise $\Delta_2^* = \Delta_1$. When $\Delta_0 > 0$, i.e. when $a < a_0 \equiv \frac{p+\eta\tilde{\Delta}-q_0\theta_{max}}{p\theta_{max}}$, then Π is bimodal. As to Δ_1^*, if we consider Δ_{opt}^1 that verifies the KKT condition for $\underline{\Pi}(\Delta) = (1-\gamma)(p-\Delta)D_3(\Delta)$, then we have $\Delta_1^* = max(0, min(\Delta_{opt}^1, \hat{\Delta}))$ and is non-increasing in a.

We can show that $\Pi_2^* - \Pi_1^*$ as a function of a has a unique zero $\underline{a} \ge a_1$ and that $\Pi_2^* - \Pi_1^* > 0$ when $a < \underline{a}$ and $\Pi_2^* - \Pi_1^* \le 0$ when $a \ge \underline{a}$. Thus, if $a < a_0$, we have $\Delta^* = 0$ iff $\bar{\Pi}'(0) \le 0$ and $\Pi_2^* - \Pi_1^* \le 0$, i.e., $a \in (max(\bar{\Pi}'^{-1}(0), \underline{a}), a_0)$. If $a \ge a_0$, then $\Delta^* = 0$ iff $\bar{\Pi}'(0) + (1-\gamma)pD_2'(0) \le 0$, i.e. there exists $a_5 > \bar{\Pi}'^{-1}(0)$ such that $a \ge max(a_0, a_5)$. Hence, $\Delta^* = 0$ iff $a > \tilde{a}_3 = max(\bar{\Pi}'^{-1}(0), \underline{a}, a_5)$.

Let $a_2 = min(\tilde{a}_2, \underline{a})$ and $a_3 = max(\underline{a}, \tilde{a}_3)$. Thus if $a < a_1$, then $\Delta^* = min(\Delta_{opt}^2, \hat{\Delta})$ and is non-increasing in a. When $a \in [a_1, a_2)$, $\Delta^* = \Delta_1$ and is non-decreasing in a. When $a \in [a_2, a_3)$, $\Delta^* \in \{min(\Delta_{opt}^2, \hat{\Delta}), min(\Delta_{opt}^1, \hat{\Delta})\}$ and is non-increasing in a. Finally, when $a \ge a_3$, $\Delta^* = 0$ and is independent of a.

(ii) We have that the optimal payoff $\Pi^* = max(\bar{\Pi}(\Delta_1), \Pi(max(0, min(\Delta_{opt}^1, \hat{\Delta}))), \bar{\Pi}(min(\Delta_{opt}^2, \hat{\Delta}))$. Using the Envelope theorem we can prove that $\bar{\Pi}(max(0, min(\Delta_{opt}^1, \hat{\Delta})))$ and $\bar{\Pi}(min(\Delta_{opt}^2, \hat{\Delta}))$ are non-decreasing in a. Similarly, we have $\Pi(\Delta_1)$ is non-decreasing in a. We have that $\Pi(max(0, min(\Delta_{opt}^1, \hat{\Delta}))) = \bar{\Pi}(min(\Delta_{opt}^1, \hat{\Delta})))$ when $a < a_3$ and $\Pi(max(0, min(\Delta_{opt}^1, \hat{\Delta}))) = \Pi(0)$ when $a \ge a_3$. We can show that $\bar{\Pi}(min(\Delta_{opt}^1, \hat{\Delta})))$ is non-decreasing in a and when $C \ge D_T(0)$, $\Pi(0) = \bar{\Pi}(0)$ is non-decreasing in a, when $C < D_T(0)$, $\Pi(0) = (p-s)(\gamma D_1 + (1-\gamma)D_3(0)) + sC$ and is non-increasing in a. There exists $a_4 \ge a_3$ such that $C \ge D_T(0)$ is equivalent to $a \le a_4$. We conclude that Π^* is non-decreasing in a for $a < a_4$ and is non-increasing in a for $a > a_4$. □

A.6. Proof of Proposition 5

(i) We have $\Delta_2^* \in \{min(\Delta_{opt}^2, \hat{\Delta}), \Delta_1\}$. We also have that Δ_1 is decreasing in t, Δ_{opt}^2 is non-increasing in t $\Delta_{opt}^2 \le \Delta_1$ and is non-decreasing in t if $\Delta_{opt}^2 > \Delta_1$ (using Topkis' theorem) and $\hat{\Delta}$ is non-decreasing in t. When $\Delta_0 > 0$, for example when $t > t_0 = \frac{z-(p+\beta/(1-\beta)\tilde{\Delta})/\theta_{max}^1}{\tilde{\Delta}}$, then Π is bimodal. As to Δ_1^*, if we consider Δ_{opt}^1 that verifies the KKT condition for $\underline{\Pi}(\Delta) = (1-\gamma)(p-\Delta)D_3(\Delta)$, then we have $\Delta_1^* = max(0, \Delta_{opt}^1)$ and is independent of t. We have $\Delta_1^* = max(0, min(\Delta_{opt}^1, \hat{\Delta}))$ and $\Delta_2^* \in \{min(\Delta_{opt}^2, \hat{\Delta}), \Delta_1\}$. We can show that $\Pi_2^* - \Pi_1^*$ as a function of t has a unique zero $\underline{t}$ and that $\Pi_2^* - \Pi_1^* < 0$ when $t < \underline{t}$ and $\Pi_2^* - \Pi_1^* \ge 0$ when $t \ge \underline{t}$.

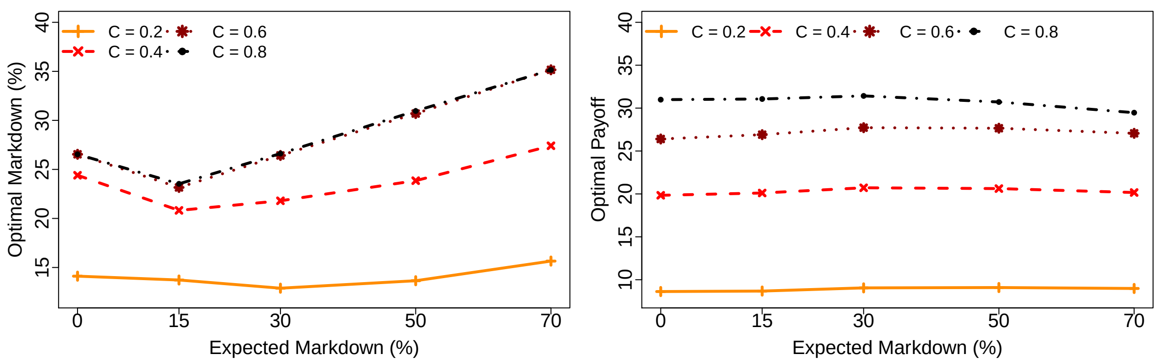
Thus if $t \le min(\underline{t}, t_0)$, then $\Delta^* = \Delta_1^*$ and is non-decreasing in t. When $t > min(\underline{t}, t_0)$, we have $\Delta^* = \Delta_2^* \in \{min(\Delta_{opt}^2, \hat{\Delta}), \Delta_1\}$. We have that $\hat{\Delta}$ is increasing in t, Δ_1 is decreasing in t and Δ_{opt}^2 is non-increasing in t when $\Delta_{opt}^2 \le \Delta_1$ and is non-decreasing in t if $\Delta_{opt}^2 > \Delta_1$. Hence, there exists $t_1 < \tilde{t} < t_2$ such that $\Delta^* = \hat{\Delta}$ and is non-decreasing when $t < t_1$, $\Delta^* = \Delta_{opt}^2 \le \Delta_1$ and is non-increasing when $t \in [t_1, \tilde{t})$, $\Delta^* = \Delta_1$ and is non-increasing when $t \in [\tilde{t}, t_2)$ and $\Delta^* = \Delta_{opt}^2 > \Delta_1$ and is non-decreasing when $t \ge t_2$. We conclude that if $t < t_1$, Δ^* is non-decreasing in t, if $t \in [t_1, t_2)$, Δ^* is non-increasing in t, if $t \ge t_2$, Δ^* is non-decreasing in t.

(ii) We have that the optimal payoff $\Pi^* = max(\Pi(\Delta_1), \Pi(max(0, min(\Delta_{opt}^1, \hat{\Delta}))), \Pi(min(\Delta_{opt}^2, \hat{\Delta}))$. Using the Envelope theorem we prove that $\Pi(max(min(\Delta_{opt}^1, \hat{\Delta}), 0))$ and $\Pi(min(\Delta_{opt}^2, \hat{\Delta}))$ are non-increasing in t. Similarly, we can show that $\Pi(\Delta_1)$ is non-increasing in t as well. We conclude that Π^* is non-increasing in t. □

Appendix B: Model Extension: Rational Expectations about Product Availability

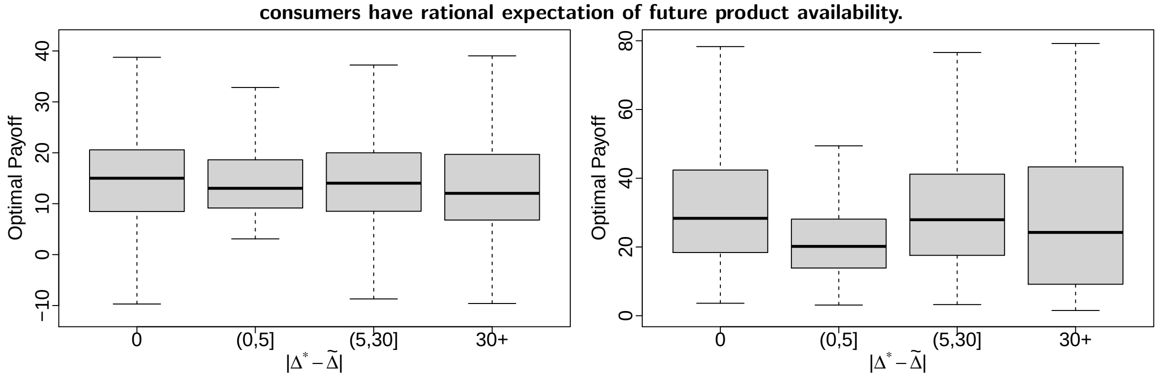
In our consumer model, we make a simplifying assumption that early consumers expect full availability of the product in Period 2 when evaluating the utility of waiting. This allows us to focus on examining quality perception without overcomplicating the analysis. To verify the robustness of our results, we numerically analyze an extension in which early consumers form rational expectations about product availability in Period 2. Specifically, let r be the actual fill rate in Period 2. That is, $r = \min\left(1, \frac{\max(C - D_1, 0)}{D_r + D_2}\right)$, where C is the initial inventory level, D_1 is the demand from early consumers who buy in Period 1, D_r is the demand from returning consumers, and D_2 is the demand from late consumers. Therefore, early consumers' utility of waiting is updated to $U(wait) = r\beta(\theta q_1 - p + \tilde{\Delta})$. Based on this updated model, we numerically analyze the retailer's optimal markdown and payoff. Figures B1–B2 closely replicate Figures 4–5, confirming that our main insights are robust to accounting for early consumers' rational expectation of product availability in Period 2. When inventory is tight (sufficient), ignoring consumers' price-based quality perception leads to 52% (15%) loss in payoff on average (see Table D5 in Supplementary Appendix D).

Figure B1 **Retailer's optimal markdown and payoff by expected markdown and inventory when early consumers have rational expectation of future product availability.**



(a) Optimal Markdown (%) (b) Optimal Payoff

Figure B2 **Retailer's Optimal Payoff versus the Difference between Optimal and Expected Markdown $\left|\Delta^* - \tilde{\Delta}\right|$ when early consumers have rational expectation of future product availability.**



(a) Low Inventory (b) High Inventory

Note. The x axis in both figures presents four cases of the absolute difference between the optimal and expected markdowns (in % points): (i) $\left|\Delta^* - \tilde{\Delta}\right| = 0$, (ii) $\left|\Delta^* - \tilde{\Delta}\right| \in (0, 5]$, (iii) $\left|\Delta^* - \tilde{\Delta}\right| \in (5, 30]$, and (iv) $\left|\Delta^* - \tilde{\Delta}\right| > 30$.

Appendix C: Detailed Results of the Online Consumer Study

C.1. Regression Analysis on Consumers' Quality Perception in Period 1

Table C1 summarizes the coefficient estimates of the various regression models we estimate for early consumers' quality perception. Independent variables include the following: p is the initial price; $\tilde{\Delta}$ is the expected markdown stated by the participants; "Male" is a dummy variable that is 1 if the participant is male.

Table C1 **Regression Estimates of Various Models on Consumers' Quality Perception in Period 1**

	Dependent variable: Early consumers' quality perception			
	(i)	(ii)	(iii)	(iv)
Intercept	53.840 (2.276)***	49.470 (2.356)***	52.460 (2.266)***	48.620 (2.341)***
p	0.171 (0.030)***	0.170 (0.029)***	0.274 (0.039)***	0.259 (0.038)***
$\tilde{\Delta}$	–	–	−0.254 (0.063)***	−0.218 (0.062)***
Male	–	8.883 (1.652)***	–	8.207 (1.644)***

Notes. Values in parentheses are the standard errors. Notation "–" means the variable does not appear in the corresponding model. ***: $p < 0.001$; p values are derived from two-sided t tests.

C.2. Regression Analysis on Consumers' Quality Perception in Period 2

In Period 2, there are two groups of consumers arriving at the market: returning consumers who were at the market in Period 1 and chose to wait for a markdown and late consumers whose first visit to the market is in Period 2. Table C2 summarizes the coefficient estimates of the various regression models we estimate for these consumers' quality perception. Compared to Table C1, we have two new independent variables: Δ is the actual markdown applied to the product in Period 2; $p - \Delta$ is the final selling price of the product.

Table C2 **Regression Estimates of Various Models on Consumers' Quality Perception in Period 2**

	Dependent variable: Consumers' quality perception in Period 2			
	(i)	(ii)	(iii)	(iv)
Intercept	56.850 (1.663)***	53.500 (1.776)***	59.430 (1.212)***	56.170 (1.372)***
p	0.204 (0.025)***	0.208 (0.025)***	–	–
Δ	−0.136 (0.032)***	−0.138 (0.032)***	–	–
$p - \Delta$	–	–	0.185 (0.024)***	0.188 (0.023)***
Male	–	5.985 (1.228)***	–	5.952 (1.232)***

Notes. Values in parentheses are the standard errors. Notation "–" means the variable does not appear in the corresponding model. ***: $p < 0.001$; p values are derived from two-sided t tests.

C.3. Model Selection: Relationships between Quality Perception and Price Information

C.3.1. Procedure. To estimate the functional relationship, we first perform a stepwise model selection that begins with a general model that contains polynomials of the relevant independent variable(s) up to the fourth degree and the square root(s) of the independent variable(s) and compares different nested versions of the general model that include different subsets of the independent variables and chooses the model with the best in-sample fit based on the Akaike Information Criterion (AIC). We then compare the best model selected by the stepwise process to a parsimonious linear model in terms of both in-sample fit and out-of-sample prediction. To do so, we perform 5-fold cross validation for each model, ensuring that the proportion of data points corresponding to each treatment condition in the study as well as key control variables (e.g., gender) is the same in each subset as in the entire data. We treat each subset as a hold-out sample, use the other four subsets as the training data to estimate the model coefficients, and predict the quality perception for the hold-out sample. We perform this estimation and prediction for each subset given a random partition, which constitutes one iteration of 5-fold validation. After one iteration, we compute the averages of a few performance measures for both in-sample fit (adjusted R^2, AIC) and out-of-sample prediction (out-of-sample adjusted R^2, mean squared errors or MSE, and mean absolute deviation or MAD). We repeat this validation for 100 iterations, using a different random partition in each iteration. Based on these 100 iterations, we finally compute the 95% confidence intervals for the above performance measures.

C.3.2. Results. Table C3 presents the coefficient estimates and model performance metrics of two candidate models for characterizing consumers' quality perception in each period. Our results show that, the simple linear models outperform or have comparable performance compared to the models obtained through the stepwise algorithm in terms of in-sample and out-of-sample predictive measures. Table F9 in Supplementary Appendix F.1 reports the results of the same procedure applied to the data from the robustness study, confirming that the simple linear models achieve comparably good in-sample and out-of-sample performance and hence, can be considered as reasonable characterization of consumers' price-based quality perception.

Table C3 Comparison of Candidate Models for Consumers' Quality Perception

	Period 1 (i)	Period 1 (ii)	Period 2 (i)	Period 2 (ii)
Intercept	48.520 (2.341)***	44.860 (6.170)***	56.170 (1.372)***	22.190 (8.926)***
p	0.259 (0.038)***	0.387 (0.199)	–	−0.762 (0.326)*
p^2	–	−0.001 (0.001)	–	−0.762 (0.326)*
$p-\Delta$	–	–	0.185 (0.024)***	–
$(p-\Delta)^4$	–	–	–	0.00000 (0.00000)
$\sqrt{p-\Delta}$	–	–	–	11.640 (3.541) **
$\tilde{\Delta}$	−0.218 (0.062)***	−0.215 (0.063)***	–	–
Δ^2	–	–	–	0.064 (0.025) **
Δ^3	–	–	–	−0.002 (0.001)*
Δ^4	–	–	–	0.00001 (0.00000)*
In-sample R^2	[0.134, 0.137]	[0.133, 0.136]	[0.098, 0.1]	[0.117, 0.119]
AIC	[1091, 1092]	[1092, 1093]	[1709, 1711]	[1707, 1709]
Out-of-sample R^2	[0.079, 0.091]	[0.064, 0.077]	[0.073, 0.082]	[0.053, 0.063]
MSE	[314.089, 321.925]	[315.385, 323.303]	[286.677, 293.513]	[282.541, 289.467]
MAD	[10.884, 11.133]	[10.915, 11.177]	[10.251, 10.434]	[10.082, 10.274]

Notes. Values in parentheses are the standard errors. Notation "–" means the variable does not appear in the corresponding model. ***: $p<0.001$; **: $p<0.01$; *: $p<0.05$; p values are derived from two-sided t tests.

C.4. Estimates of Gain/Loss Utility Parameters from the Consumer Study

Table C4 summarizes the structural estimation results for returning consumers' gain/loss utility parameters based on the data from our consumer study and the formulation discussed in §5.2.4. These results suggest that returning consumers exhibit a significant loss emotion but do not show a sign of significant gain emotion.

Table C4 Comparison of Candidate Utility Models for Returning Consumers

	Full	$\eta_1=0$	$\eta_1=\eta_2\geq 0$	$\eta_1=\eta_2=0$
a	0	0	0	0
b	3.9	3.9	2.8	2.45
η_1	0	–	0.49	–
η_2	1.75	1.75	0.49	–
$\mathcal{LL}$	-120.98	-120.98	-122.80	-127.07
Likelihood ratio test		$p=1.00$	$p=0.034$	$p=0.001$